Единственность решения системы уравнений для индукции магнитного поля

$$\operatorname{rot}\mathbf{B} = \mu_0\mathbf{j} \qquad \operatorname{div}\mathbf{B} = 0$$

$$\mathbf{B}_{2t} - \mathbf{B}_{1t} = \mathbf{i} \qquad \mathbf{B}_{2n} = \mathbf{B}_{1n}$$

$$\mathbf{B} - \mathbf{B}' = \mathbf{B}''$$

$$\operatorname{rot}\mathbf{B}'' = 0 \qquad \operatorname{div}\mathbf{B}'' = 0 \qquad \mathbf{B}''_{2t} - \mathbf{B}''_{1t} = 0 \qquad \mathbf{B}''_{2n} = \mathbf{B}''_{1n}$$

$$\operatorname{div}[\mathbf{a}\times\mathbf{b}] = \mathbf{b}\operatorname{rot}\mathbf{a} - \mathbf{a}\operatorname{rot}\mathbf{b}$$

$$\int \mathbf{b}\operatorname{rot}\mathbf{a}\,dv = \int \operatorname{div}[\mathbf{a}\times\mathbf{b}]dv + \int \mathbf{a}\operatorname{rot}\mathbf{b}\,dv$$

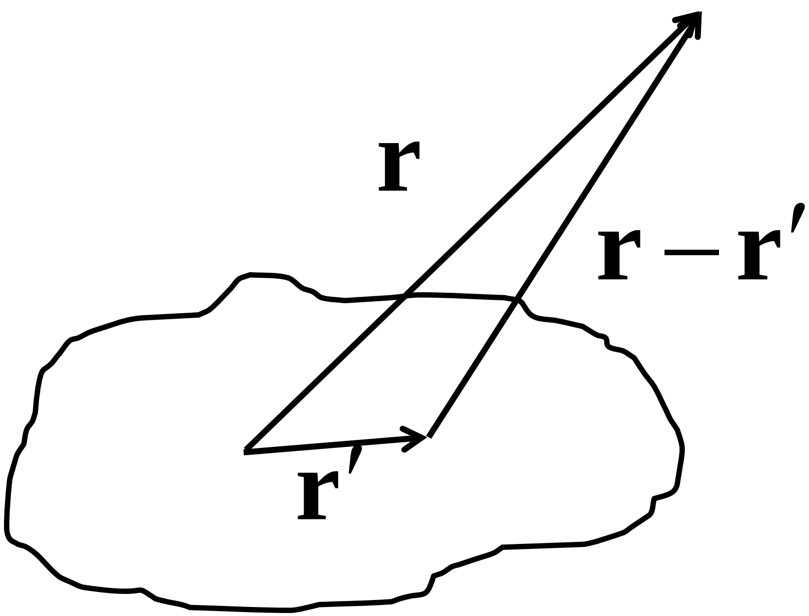
$$\mathbf{b} = \mathbf{B}'' \qquad \mathbf{a} = \mathbf{A}'' \qquad = 0 \qquad \mathbf{B}'' = \operatorname{rot}\mathbf{A}''$$

$$\int_V \mathbf{B}''\cdot\mathbf{B}''dv = \int_V \operatorname{div}[\mathbf{A}''\times\mathbf{B}'']dv = \int_S [\mathbf{A}''\times\mathbf{B}'']_n\,dS \Rightarrow 0$$

Векторный потенциал элементарного тока на больших расстояниях

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int\limits_V \frac{\mathbf{j}(\mathbf{r}')d\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}$$

$$\frac{1}{r-r'} \approx \frac{1}{r} + \frac{(\mathbf{r}'\cdot\mathbf{r})}{r^3} + \dots$$



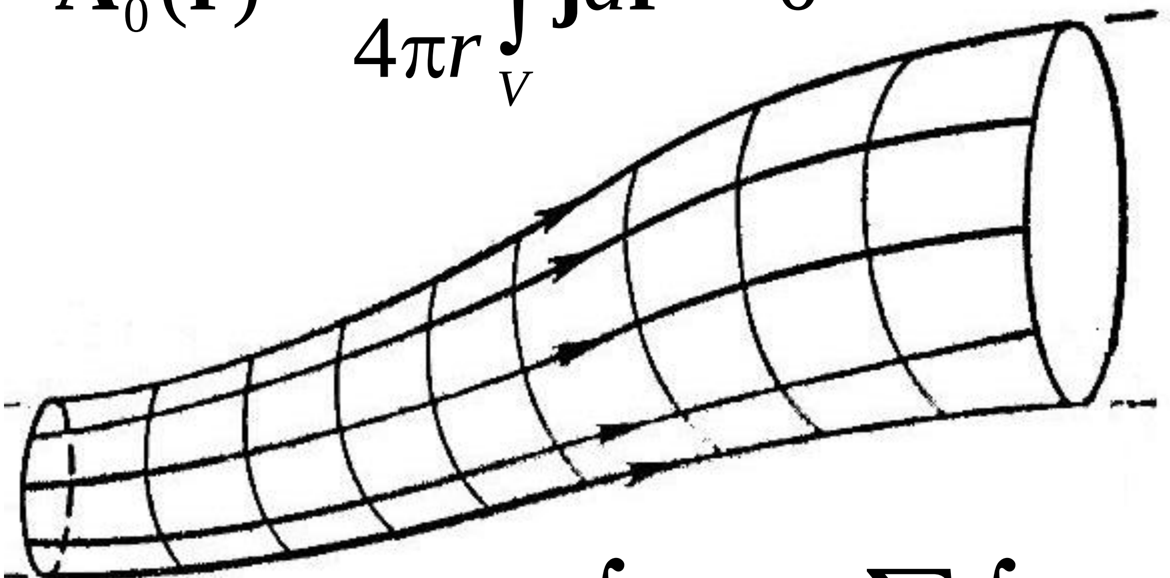
$$\mathbf{A}(\mathbf{r}) = \mathbf{A}_0(\mathbf{r}) + \mathbf{A}_1(\mathbf{r}) + \ldots =$$

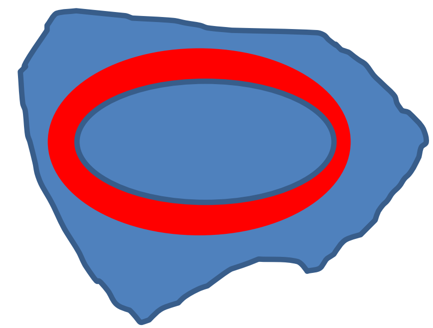
$$= \frac{\mu_0}{4\pi r} \int\limits_V \mathbf{j}\,d\mathbf{r}' +$$

$$+ \frac{\mu_0}{4\pi r^3} \int\limits_V \mathbf{j}(\mathbf{r}'\cdot\mathbf{r})d\mathbf{r}' + \ldots$$

$$\mathbf{A}_0(\mathbf{r}) = \frac{\mu_0}{4\pi r}\int_V \mathbf{j}\,d\mathbf{r}' = 0$$

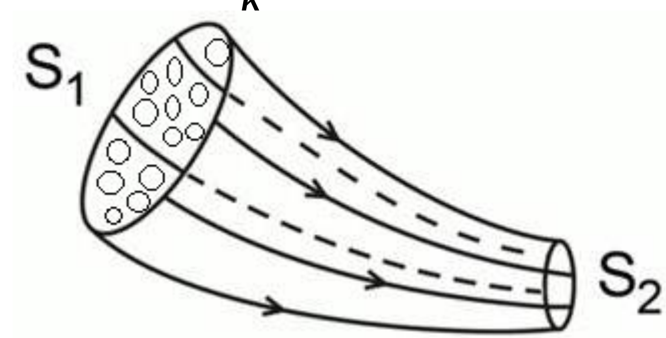
$$\operatorname{div}\mathbf{j} = 0$$





$$\int_V \mathbf{j}\,d\mathbf{r}' = \sum_k \int_{L_k} I_k\,d\mathbf{l}_k = \sum_k I_k \oint d\mathbf{l}_k = 0$$

$$\mathbf{A} = \mathbf{A}_1(\mathbf{r}) = \frac{\mu_0}{4\pi r^3}\int_V \mathbf{j}(\mathbf{r}'\cdot\mathbf{r})\,d\mathbf{r}'$$



$$\mathbf{j}(\mathbf{r}'\cdot\mathbf{r}) = \frac{1}{2}\{\mathbf{j}(\mathbf{r}'\cdot\mathbf{r}) - \mathbf{r}'(\mathbf{j}\cdot\mathbf{r})\} + \frac{1}{2}\{\mathbf{j}(\mathbf{r}'\cdot\mathbf{r}) + \mathbf{r}'(\mathbf{j}\cdot\mathbf{r})\}$$

$$\mathbf{j}(\mathbf{r}'\cdot\mathbf{r})-\mathbf{r}'(\mathbf{j}\cdot\mathbf{r})=-[\mathbf{r}\times[\mathbf{r}'\times\mathbf{j}]]=[[\mathbf{r}'\times\mathbf{j}]\times\mathbf{r}]$$

$$\mathbf{A}_1(\mathbf{r})=\frac{\mu_0}{8\pi r^3}\int_V[[\mathbf{r}'\times\mathbf{j}(\mathbf{r}')]\times\mathbf{r}]d\mathbf{r}'+$$

$$+\frac{\mu_0}{8\pi r^3}\int_V\{\mathbf{j}(\mathbf{r}'\cdot\mathbf{r})+\mathbf{r}'(\mathbf{j}\cdot\mathbf{r})\}d\mathbf{r}'$$

Покажем, что второй интеграл равен нулю

$$\mathbf{a}\cdot\int_V\{\mathbf{j}(\mathbf{r}'\cdot\mathbf{r})+\mathbf{r}'(\mathbf{j}\cdot\mathbf{r})\}d\mathbf{r}'=$$

$$=\int_V\{(\mathbf{a}\cdot\mathbf{j})(\mathbf{r}'\cdot\mathbf{r})+(\mathbf{a}\cdot\mathbf{r}')(\mathbf{j}\cdot\mathbf{r})\}d\mathbf{r}'=$$

$$=\int_V\{\mathbf{j}\,\mathrm{grad}'(\mathbf{a}\cdot\mathbf{r}')(\mathbf{r}'\cdot\mathbf{r})+(\mathbf{a}\cdot\mathbf{r}')\mathbf{j}\cdot\mathrm{grad}'(\mathbf{r}'\cdot\mathbf{r})\}d\mathbf{r}'=$$

$$=\int_V\mathbf{j}\cdot\mathrm{grad}'\{(\mathbf{a}\cdot\mathbf{r}')(\mathbf{r}'\cdot\mathbf{r})\}d\mathbf{r}' \Rightarrow$$

$$\mathrm{div}'(\gamma \times \mathbf{b}) = \gamma\,\mathrm{div}'\mathbf{b} + \mathbf{b}\,\mathrm{grad}'\gamma$$

$$\Rightarrow \int_V \mathbf{j}\cdot\mathrm{grad}'\{(\mathbf{a}\cdot\mathbf{r}')(\mathbf{r}'\cdot\mathbf{r})\}d\mathbf{r}' = \int_V \mathrm{div}'\{(\mathbf{a}\cdot\mathbf{r}')(\mathbf{r}'\cdot\mathbf{r})\mathbf{j}\}d\mathbf{r}' -$$

$$-\int_V \underbrace{(\mathbf{a}\cdot\mathbf{r}')(\mathbf{r}'\cdot\mathbf{r})\,\mathrm{div}'\,\mathbf{j}\,d\mathbf{r}'}_{=0} = \int_S \underbrace{j_n (\mathbf{a}\cdot\mathbf{r}')(\mathbf{r}'\cdot\mathbf{r})dS}_{=0} = 0$$

$$(\mathrm{div}'\,\mathbf{j} = 0) \qquad (j_n = 0)$$

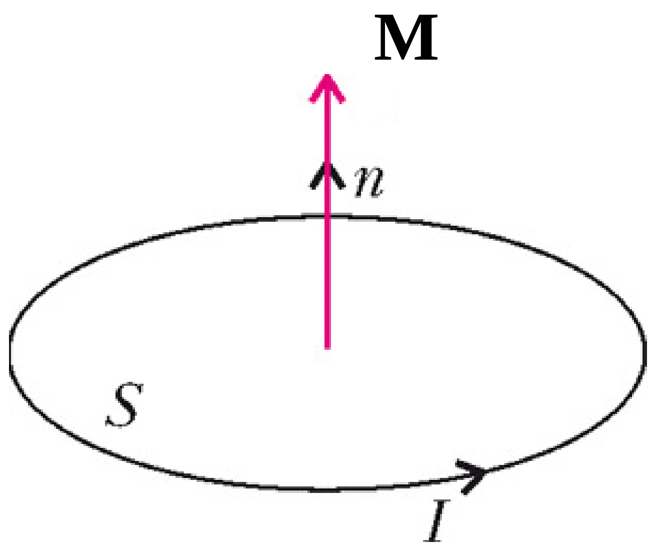
$\mathbf{a}$ - произвольный вектор $\Rightarrow \displaystyle\int_V \{\mathbf{j}(\mathbf{r}'\cdot\mathbf{r}) + \mathbf{r}'(\mathbf{j}\cdot\mathbf{r})\}d\mathbf{r}' = 0$

$$\mathbf{A}_1(\mathbf{r}) = \frac{\mu_0}{8\pi r^3}\int_V [[\mathbf{r}'\times\mathbf{j}(\mathbf{r}')]\times\mathbf{r}]d\mathbf{r}' = \frac{\mu_0}{4\pi r^3}\int_V [\mathbf{J}(\mathbf{r}')\times\mathbf{r}]d\mathbf{r}' =$$

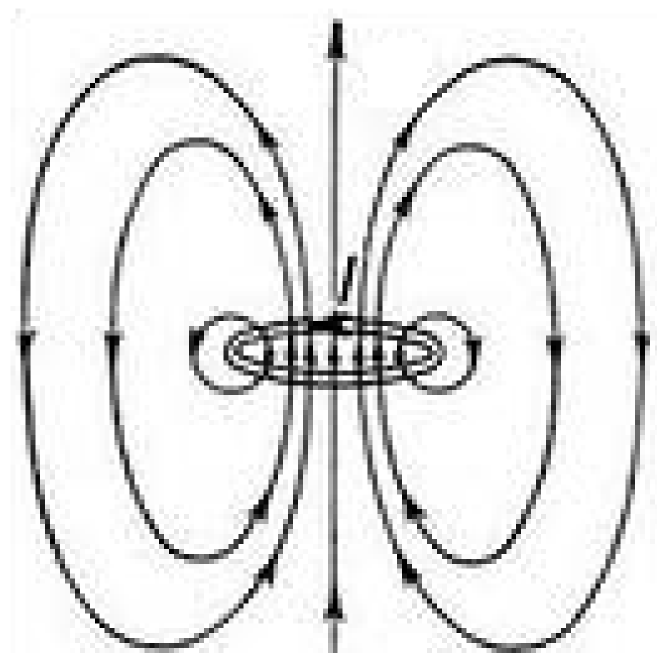
$$= \frac{\mu_0}{4\pi}\frac{[\mathbf{M}\times\mathbf{r}]}{r^3}$$

$$\mathbf{J}(\mathbf{r}') = \frac{1}{2}[\mathbf{r}'\times\mathbf{j}(\mathbf{r}')] \qquad \mathbf{M} = \int_V \mathbf{J}(\mathbf{r}')d\mathbf{r}' = \frac{1}{2}\int_V [\mathbf{r}'\times\mathbf{j}]d\mathbf{r}'$$

$$\mathbf{M} = \int_V \mathbf{J}(\mathbf{r}')d\mathbf{r}' = \frac{1}{2}\int_V [\mathbf{r}' \times \mathbf{j}]d\mathbf{r}' = \frac{I}{2}\int_L [\mathbf{r}' \times d\mathbf{l}]$$



$$\mathbf{M} = IS\mathbf{n}$$



Индукция магнитного поля на больших расстояниях равна

$$\mathbf{B} = \frac{\mu_0}{4\pi}\left\{\frac{3\mathbf{r}(\mathbf{M}\mathbf{r})}{r^5} - \frac{\mathbf{M}}{r^3}\right\}$$

Магнитное поле совокупности элементарных токов на больших расстояниях

$$\mathbf{B} = \operatorname{rot}\mathbf{A} = \frac{\mu_0}{4\pi}\operatorname{rot}\frac{[\mathbf{M}\times\mathbf{r}\}}{r^3} =$$

$$= \frac{\mu_0}{4\pi}\left\{\frac{1}{r^3}\operatorname{rot}[\mathbf{M}\times\mathbf{r}] + \left[\operatorname{grad}\left(\frac{1}{r^3}\right)\times[\mathbf{M}\times\mathbf{r}]\right]\right\}$$

$$\operatorname{rot}[\mathbf{M}\times\mathbf{r}] = \underset{=0}{(\mathbf{r}\operatorname{grad})\mathbf{M}} - \underset{=\mathbf{M}}{(\mathbf{M}\operatorname{grad})\mathbf{r}} + \underset{=3\mathbf{M}}{\mathbf{M}\operatorname{div}\mathbf{r}} - \underset{=0}{\mathbf{r}\operatorname{div}\mathbf{M}} = 2\mathbf{M}$$

$$\left[\operatorname{grad}\left(\frac{1}{r^3}\right)\times[\mathbf{M}\times\mathbf{r}]\right] = -\frac{3}{r^5}[\mathbf{r}\times[\mathbf{M}\times\mathbf{r}]] = -\frac{3\mathbf{M}}{r^3} + \frac{3\mathbf{r}(\mathbf{M}\cdot\mathbf{r})}{r^5}$$

$$\mathbf{B} = \frac{\mu_0}{4\pi}\left\{\frac{3\mathbf{r}(\mathbf{M}\mathbf{r})}{r^5} - \frac{\mathbf{M}}{r^3}\right\}$$

Магнитное поле в среде

$$\operatorname{rot}\mathbf{B}_m = \mu_0 \mathbf{j}_m = \mu_0 \rho \mathbf{v}_m$$

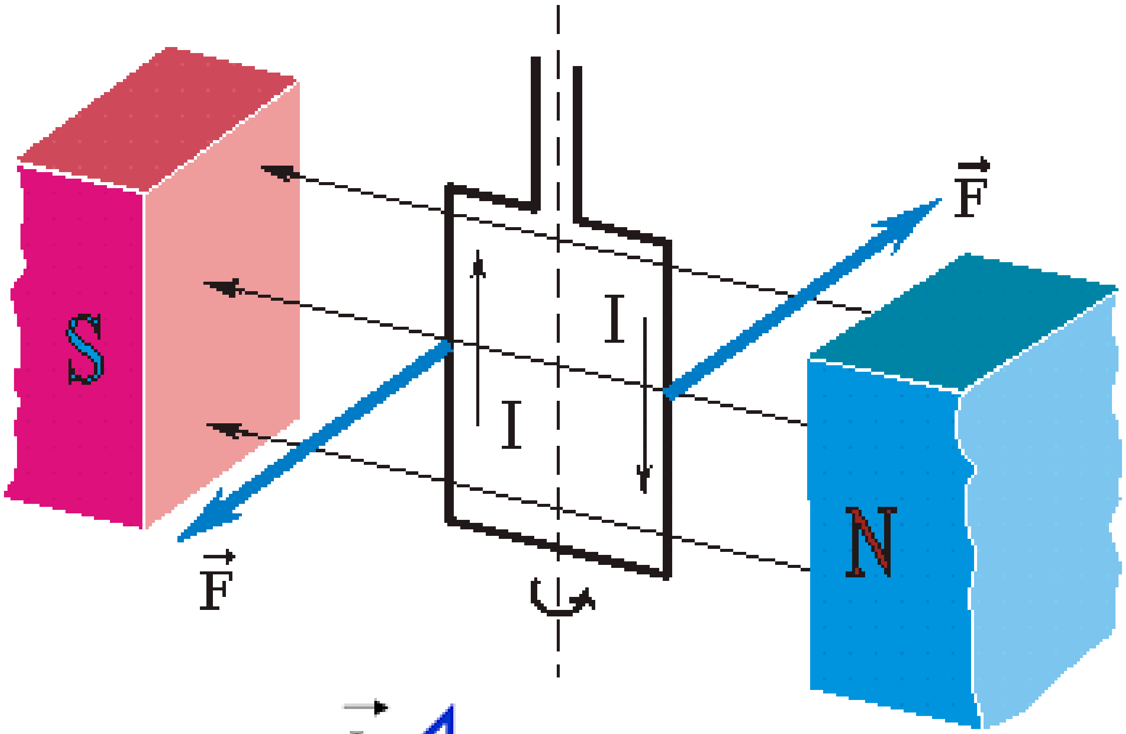
$$\operatorname{div}\rho\mathbf{v}_m = 0 \qquad \operatorname{div}\mathbf{B}_m = 0$$

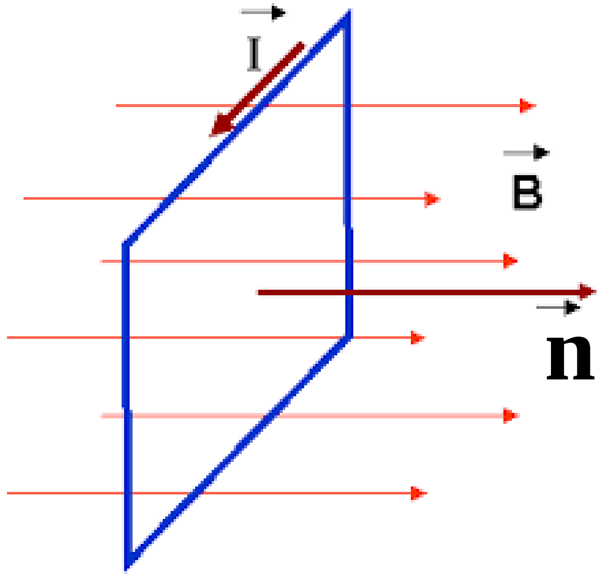
$$\overline{f} = \frac{1}{2\tau}\int_{-\tau}^{\tau}\frac{1}{v_0}\int_{v_0} f_m dv dt$$

$$\operatorname{rot}\mathbf{B} = \mu_0 \overline{\rho\mathbf{v}_m} \qquad \operatorname{div}\mathbf{B} = 0 \qquad \operatorname{div}\overline{\rho\mathbf{v}_m} = 0$$

$$\overline{\rho\mathbf{v}_m} = \mathbf{j} + \mathbf{j}_\mathrm{m}$$

Элементарный ток в магнитном поле

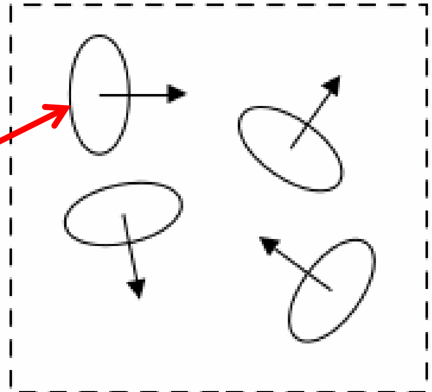


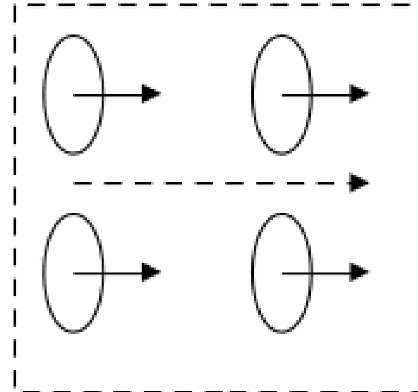


$$\mathbf{B} \uparrow\uparrow \mathbf{M} = IS\mathbf{n}$$

Молекулярный ток

$\mathbf{j}_m$



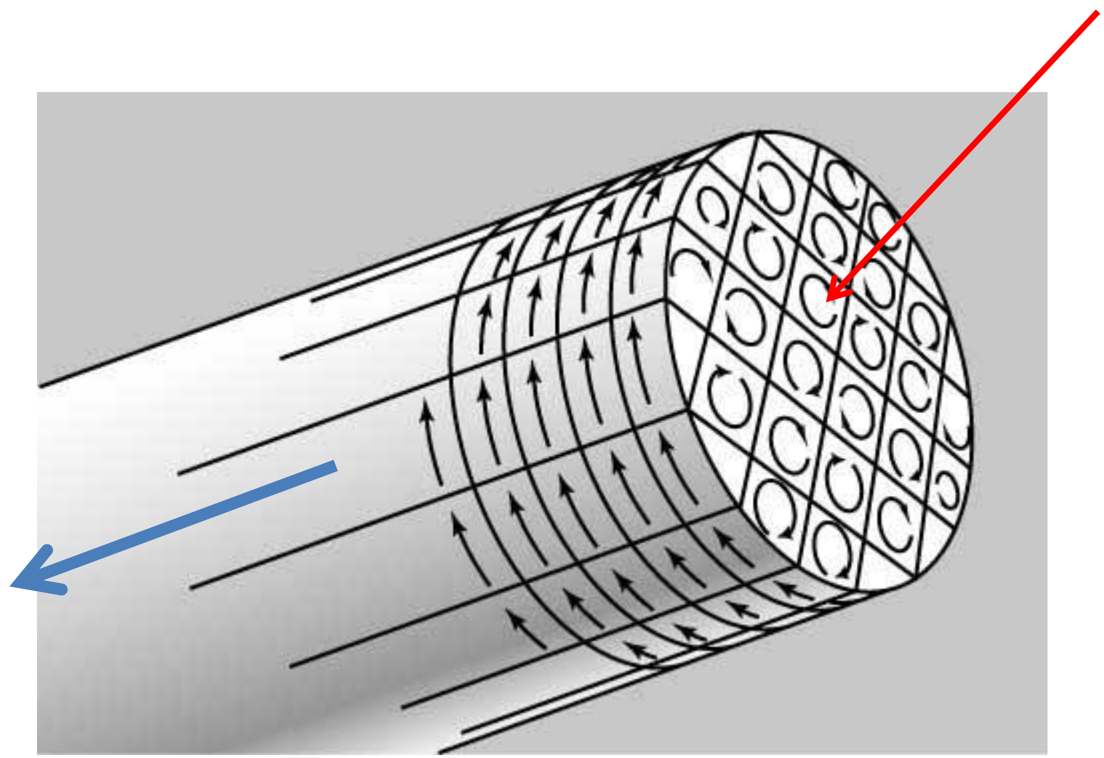


$$\mathbf{M} = \int_V \mathbf{J}(\mathbf{r}')d\mathbf{r}' =$$

$$= \frac{1}{2}\int_V [\mathbf{r}' \times \mathbf{j}]d\mathbf{r}'$$

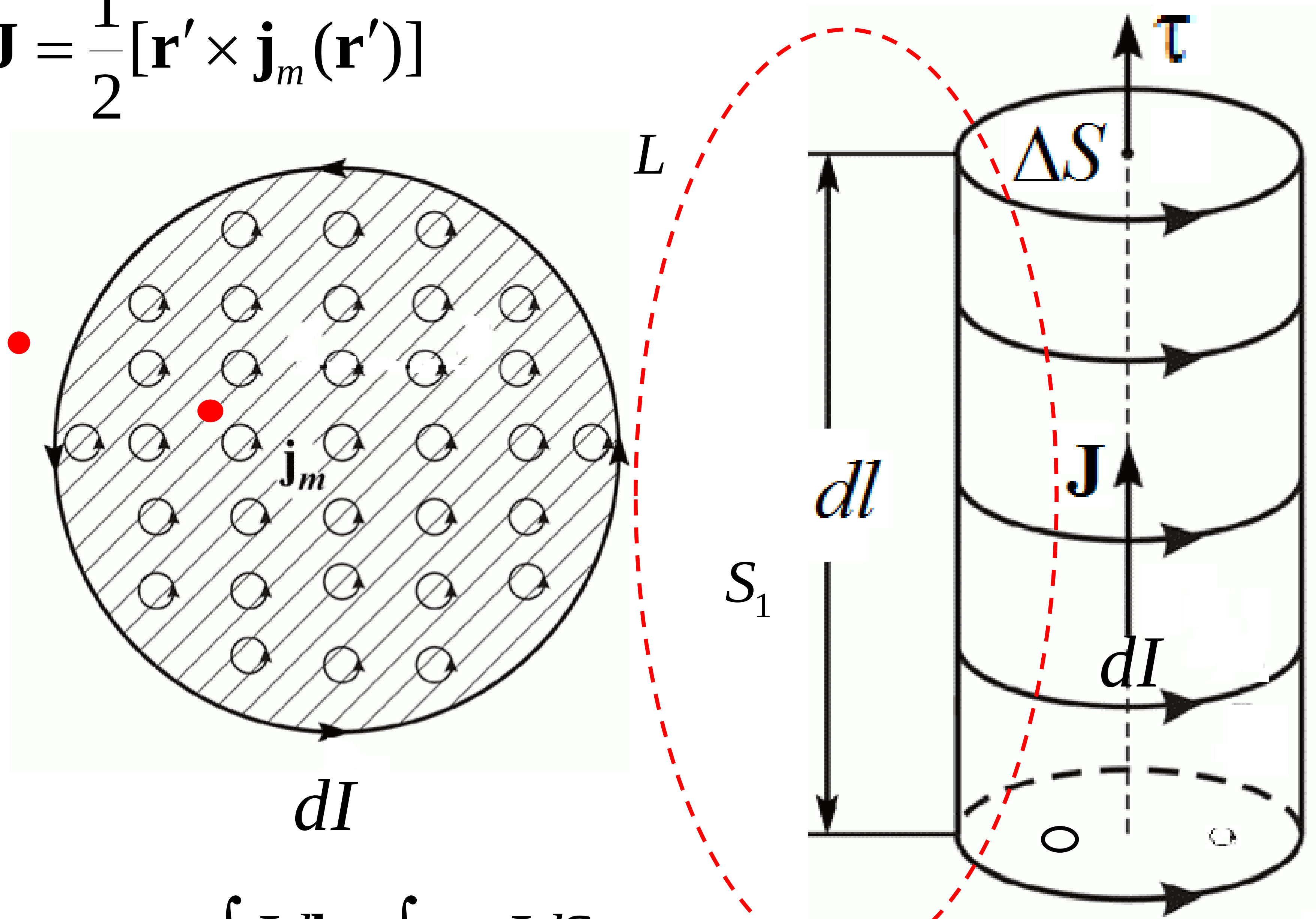
$$\mathbf{M} = \int \mathbf{J}dv$$

$\mathbf{j}_m$

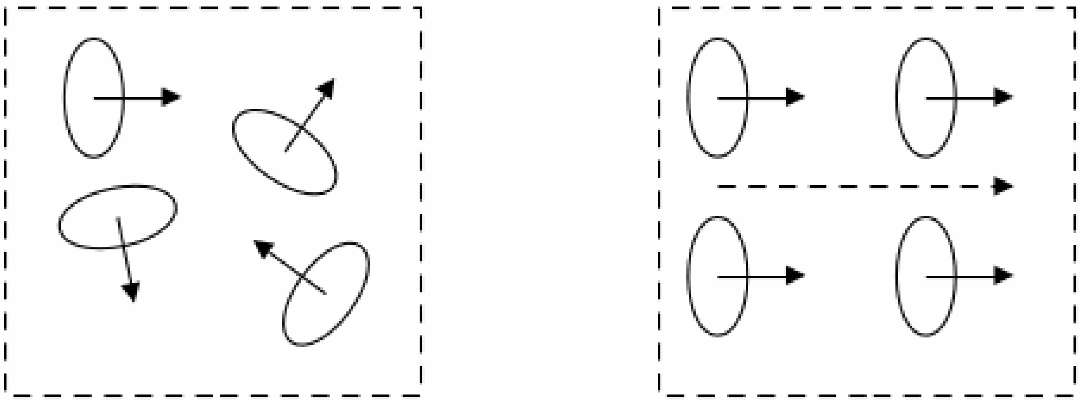


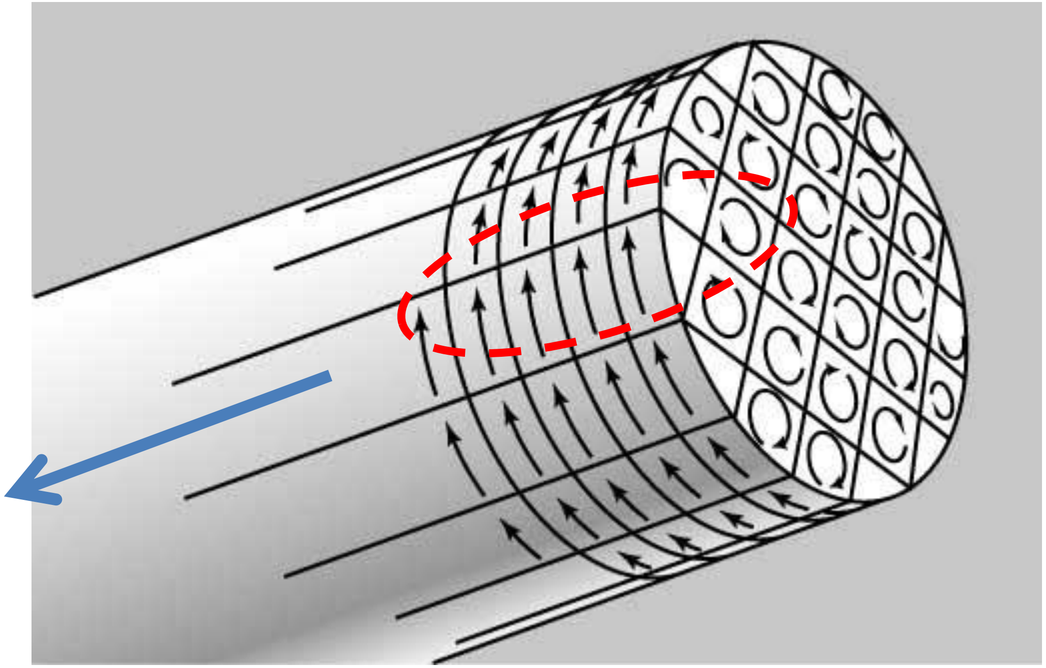
J –плотность дипольного магнитного момента

$$\mathbf{J} = \frac{1}{2}[\mathbf{r}' \times \mathbf{j}_m(\mathbf{r}')]$$



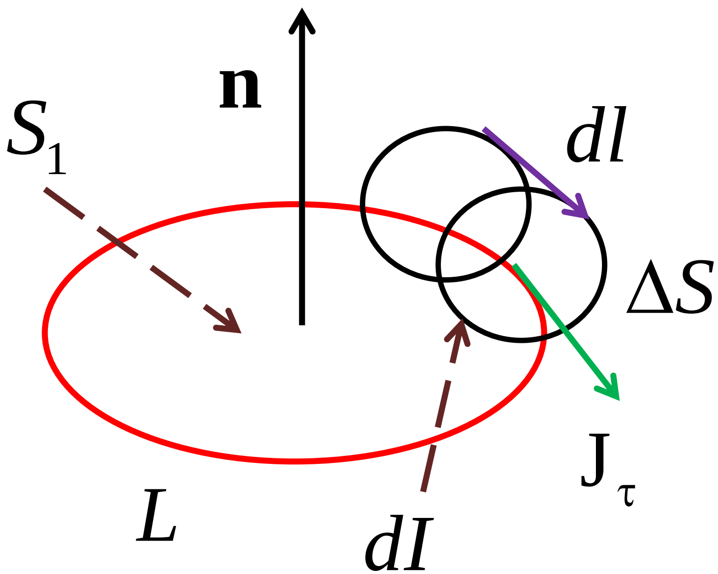
$$\int_L \mathbf{J} d\mathbf{l} = \int_{S_1} \operatorname{rot} \mathbf{J} d\mathbf{S}_1$$





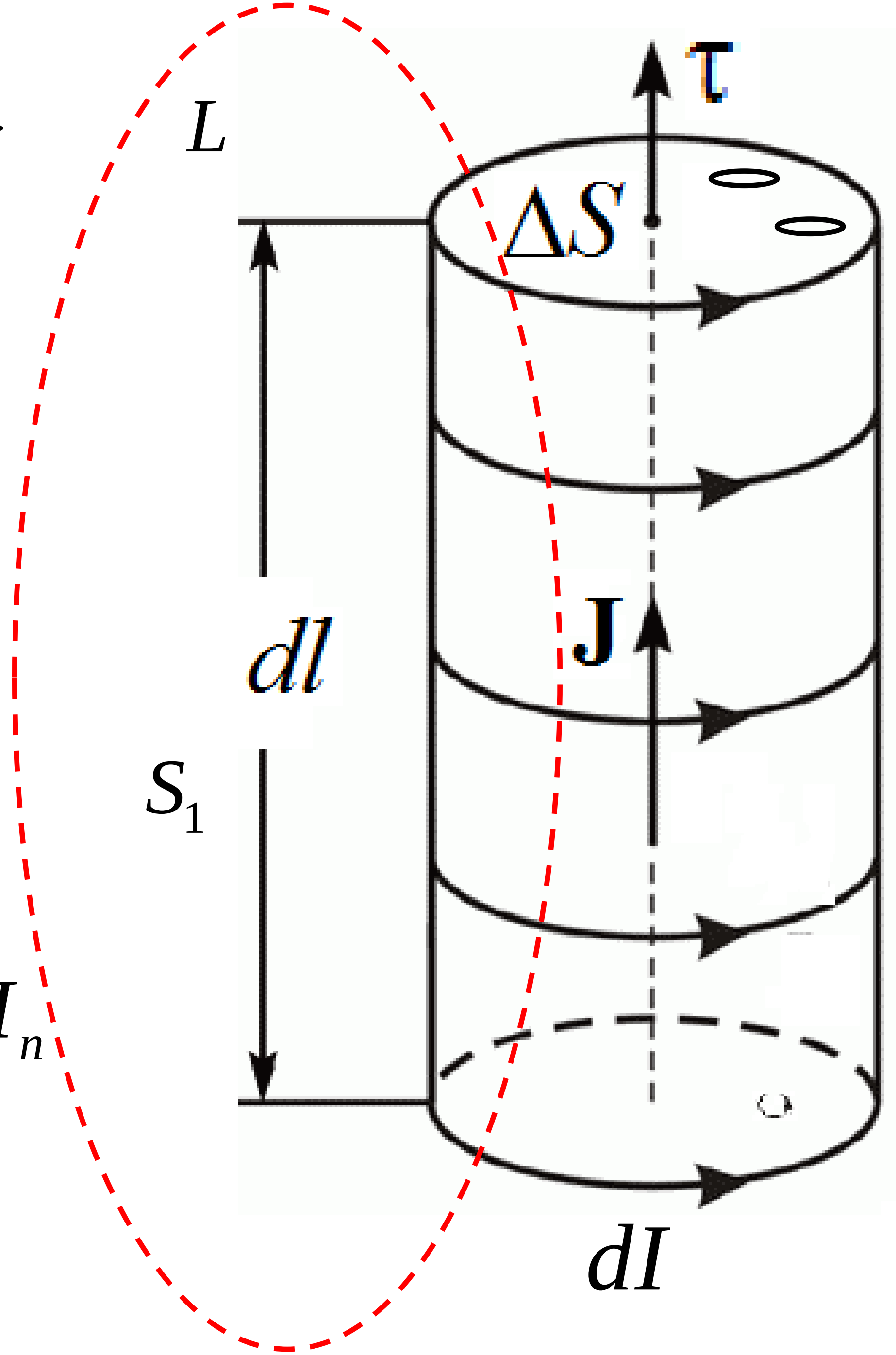
$$\mathbf{M} = \int \mathbf{J} dv$$

$$\int_L \mathbf{J}d\mathbf{l} = \int_L \mathrm{J}_\tau\, dl \frac{\Delta S}{\Delta S} = \int_L \mathrm{J}_\tau \frac{dV}{\Delta S} \Rightarrow$$



$$\Rightarrow \int_L \frac{dM_\tau}{\Delta S} = \int_L \frac{dI_n \Delta S}{\Delta S} = \int_L dI_n$$

$$= I = \int_{S_1} \mathbf{j}_m d\mathbf{S}_1 = \int_{S_1} \mathrm{rot}\, \mathbf{J} d\mathbf{S}_1$$



$$\boxed{\mathbf{j}_m = \operatorname{rot}\mathbf{J}}$$

$$\operatorname{rot}\mathbf{B} = \mu_0(\mathbf{j} + \mathbf{j}_m) = \mu_0\mathbf{j} + \mu_0 \operatorname{rot}\mathbf{J}$$

$$\mathbf{H} = \mathbf{B}/\mu_0 - \mathbf{J} = \mathbf{B}/\mu_0 - \alpha\mathbf{B} \qquad \mu = (1 - \alpha\mu_0)^{-1}$$

$$\operatorname{rot}\mathbf{H} = \mathbf{j} \qquad \operatorname{div}\mathbf{B} = 0 \qquad \mathbf{B} = \mu\mu_0\mathbf{H}$$

Граничные условия

$$\mathbf{H}_{2t} - \mathbf{H}_{1t} = \mathbf{i} \qquad \mathbf{B}_{2n} - \mathbf{B}_{1n} = 0$$

Единственность решения системы уравнений для индукции и напряженности магнитного поля

$$\mathbf{B} - \mathbf{B}' = \mathbf{B}'' \qquad \mathbf{H} - \mathbf{H}' = \mathbf{H}''$$

Единственность решения системы уравнений для индукции магнитного поля

$$\mathrm{rot}\,\mathbf{H}'' = 0 \qquad \mathrm{div}\,\mathbf{B}'' = 0 \qquad \mathbf{H}''_{2t} = \mathbf{H}''_{1t} \qquad \mathbf{B}''_{2n} = \mathbf{B}''_{1n}$$

$$\mathrm{div}[\mathbf{a}\times\mathbf{b}] = \mathbf{b}\,\mathrm{rot}\,\mathbf{a} - \mathbf{a}\,\mathrm{rot}\,\mathbf{b}$$

$$\int \mathbf{b}\,\mathrm{rot}\,\mathbf{a}\,dv = \int \mathrm{div}[\mathbf{a}\times\mathbf{b}]\,dv + \int \mathbf{a}\,\mathrm{rot}\,\mathbf{b}\,dv$$

$$\mathbf{b} = \mathbf{H}'' \qquad \mathbf{a} = \mathbf{A}'' \qquad \mathbf{B}'' = \mathrm{rot}\,\mathbf{A}''$$

$$\int_V \mathbf{H}''\cdot\mathbf{B}''\,dv = \frac{1}{\mu\mu_0}\int_V \mathrm{div}[\mathbf{A}''\times\mathbf{B}'']\,dv + \underbrace{\int_V \mathbf{A}''\,\mathrm{rot}\,\mathbf{H}''\,dv}_{=0} =$$

$$= \frac{1}{\mu\mu_0}\int_S \underbrace{[\mathbf{A}''\times\mathbf{B}'']_n}_{\sim 1/r^3}\,dS \Rightarrow 0$$

$$\Delta\mathbf{A} = -\mu_0(\mathbf{j} + \mathbf{j}_m) = -\mu_0\mathbf{j} - \mu_0 \operatorname{rot}\mathbf{J} =$$

$$= -\mu_0\mathbf{j} - \alpha\mu_0 \operatorname{rot}\mathbf{B} = -\mu_0\mathbf{j} - \alpha\mu_0 \operatorname{rotrot}\mathbf{A} =$$

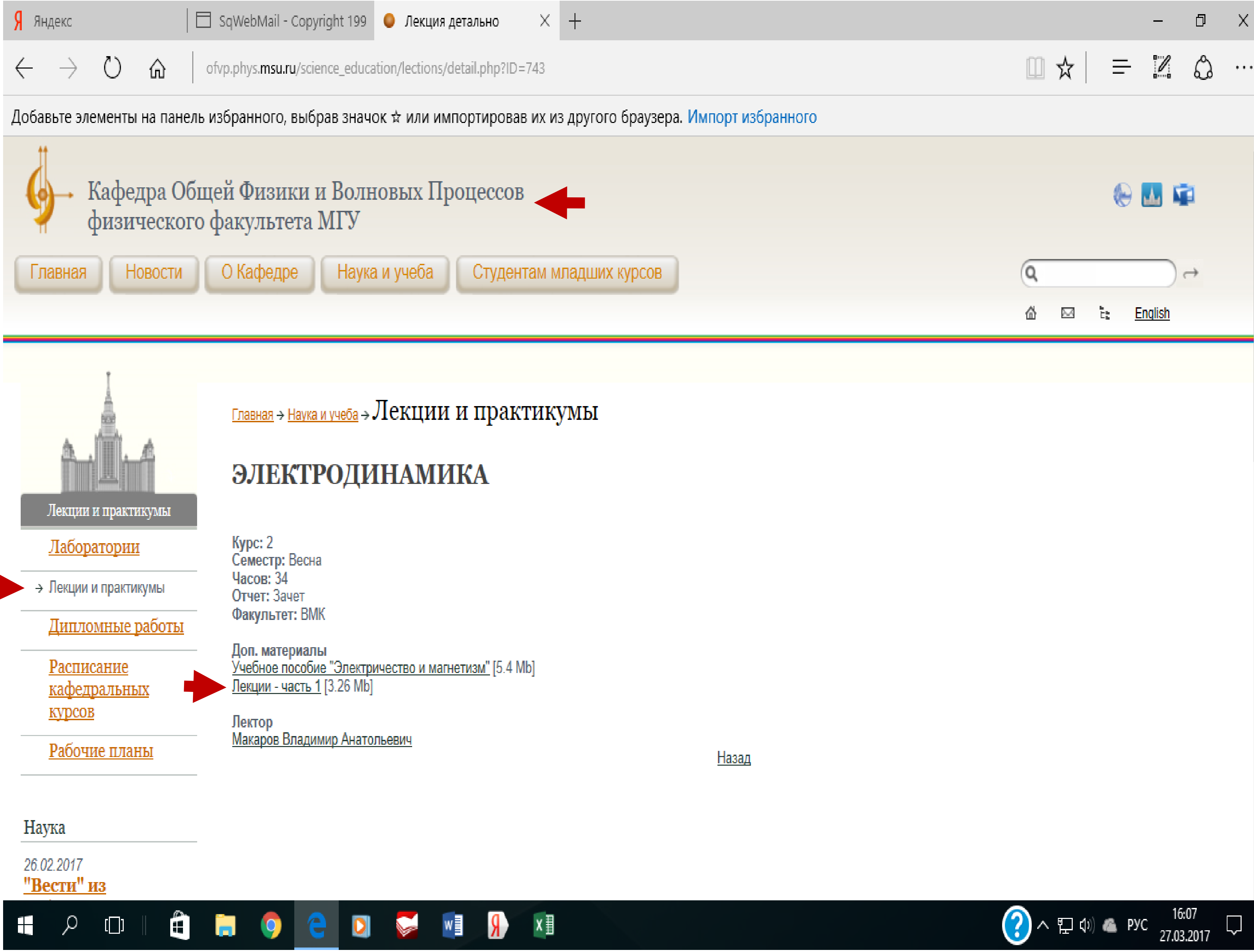
$$-\mu_0\mathbf{j} + \alpha\mu_0\Delta\mathbf{A} \qquad\qquad \operatorname{div}\mathbf{A} = 0$$

Уравнение Пуассона

$$\Delta\mathbf{A} = -\mu_0\mu\mathbf{j} \quad \Longrightarrow \quad \mathbf{A}(\mathbf{r}) = \frac{\mu_0\mu}{4\pi}\int_V \frac{\mathbf{j}(\mathbf{r}')d\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}$$

$$\mu = (1-\alpha\mu_0)^{-1}$$

$$\mathbf{B}(\mathbf{r}) = \operatorname{rot}\mathbf{A}(\mathbf{r}) = \frac{\mu_0\mu}{4\pi}\int_V \frac{[\mathbf{j}(\mathbf{r}')\times(\mathbf{r}-\mathbf{r}')]d\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}$$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"Вести" из
Главная → Наука и учеба → Лекции и практикумы
ЭЛЕКТРОДИНАМИКА
Курс: 2
Семестр: Весна
Часов: 34
Отчет: Зачет
Факультет: ВМК
Доп. материалы
Учебное пособие "Электричество и магнетизм" [5.4 Mb]
Лекции - часть 1 [3.26 Mb]
Лектор
Макаров Владимир Анатольевич
Назад
РУС
16:07
27.03.2017

Сила действующая на элементарный ток

$$\mathbf{F} = \int_V [\mathbf{j} \times \mathbf{B}]\, dv$$

$$\mathbf{F} = \int_V [\mathbf{j} \times \mathbf{B}_0]\, dv = -\left[\mathbf{B}_0 \times \overbrace{\int_V \mathbf{j}\, dv}^{=0}\right] = 0$$

$$B_i = B_i(0) + x\left(\frac{\partial B_i}{\partial x}\right)_0 + y\left(\frac{\partial B_i}{\partial y}\right)_0 + z\left(\frac{\partial B_i}{\partial z}\right)_0$$

$$F_z = \int_V j_x \left(x\left(\frac{\partial B_y}{\partial x}\right)_0 + y\left(\frac{\partial B_y}{\partial y}\right)_0 + z\left(\frac{\partial B_y}{\partial z}\right)_0 \right) dv -$$

$$- \int_V j_y \left(x\left(\frac{\partial B_x}{\partial x}\right)_0 + y\left(\frac{\partial B_x}{\partial y}\right)_0 + z\left(\frac{\partial B_x}{\partial z}\right)_0 \right) dv =$$

Сила действующая на элементарный ток

$$F_z = \left(\frac{\partial B_y}{\partial x}\right)_0 \cdot \int\limits_V \underline{j_x x dv +} \left(\frac{\partial B_y}{\partial y}\right)_0 \cdot \int\limits_V j_x y dv +$$

$$+\left(\frac{\partial B_y}{\partial z}\right)_0 \cdot \int\limits_V j_x z dv - \left(\frac{\partial B_x}{\partial x}\right)_0 \cdot \int\limits_V j_y x dv -$$

$$-\left(\frac{\partial B_x}{\partial y}\right)_0 \cdot \int\limits_V \underline{j_y y dv -} \left(\frac{\partial B_x}{\partial z}\right)_0 \cdot \int\limits_V j_y z dv$$

$$\int\limits_V j_y y dv = \int\limits_V j_x x dv = 0$$

$$\varphi = x^2 / 2$$

$$\operatorname{div}(\varphi\mathbf{a}) = \varphi \operatorname{div}\mathbf{a} + \mathbf{a}\operatorname{grad}\varphi$$

$$\mathbf{a} = \mathbf{j}$$

$$x j_x = \frac{1}{2}\operatorname{div}(x^2\mathbf{j}) - \frac{x^2}{2}\operatorname{div}\mathbf{j} = \frac{1}{2}\operatorname{div}(x^2\mathbf{j})$$

$$\int_V j_x x dv = \frac{1}{2}\int_V \mathrm{div}(x^2\mathbf{j})dv = \frac{1}{2}\oint_{S_{провод.}} x^2 j_n dS = 0$$

$$\int_V j_y y dv = 0 \qquad\qquad \mathbf{M} = \frac{1}{2}\int_V [\mathbf{r}\times\mathbf{j}]d\mathbf{r}$$

$$\int_V x j_y dv = \frac{1}{2}\int_V \underbrace{(xj_y - yj_x)dv}_{= M_z} + \frac{1}{2}\int_V \underbrace{(xj_y + yj_x)dv}_{= 0} = M_z$$

$$xj_y + yj_x = \mathbf{j}\,\mathrm{grad}(xy) \qquad\qquad \mathrm{div}(\varphi\mathbf{a}) = \varphi\,\mathrm{div}\,\mathbf{a} + \mathbf{a}\,\mathrm{grad}\,\varphi$$

$$\int_V (xj_y + yj_x)dv = \int_V \mathbf{j}\,\mathrm{grad}(xy)dv =$$

$$= \int_V \mathrm{div}(xy\mathbf{j})dv - \int_V xy\,\underbrace{\mathrm{div}\,\mathbf{j}\,dv}_{= 0\ (\mathrm{div}\,\mathbf{j} = 0)} = \int_{S_{провод.}} xyj_n dS = 0$$

Аналогично $$\int_V y j_x dv = -M_z$$

$$\int_V y j_z dv = -\int_V z j_y dv = M_x \qquad \int_V z j_x dv = -\int_V x j_z dv = M_y$$

$$F_z = -M_z\left(\frac{\partial B_y}{\partial y}\right) + M_y\left(\frac{\partial B_y}{\partial z}\right) - M_z\left(\frac{\partial B_x}{\partial x}\right) + M_x\left(\frac{\partial B_x}{\partial z}\right) =$$
$$= M_x\left(\frac{\partial B_x}{\partial z}\right) + M_y\left(\frac{\partial B_y}{\partial z}\right) + M_z\left(\frac{\partial B_z}{\partial z}\right) = \frac{\partial}{\partial z}(\mathbf{MB})$$

$$\boxed{\mathbf{F} = \mathrm{grad}(\mathbf{MB})}$$

$$F_z = M_x\left(\frac{\partial B_x}{\partial z}\right) + M_y\left(\frac{\partial B_y}{\partial z}\right) + M_z\left(\frac{\partial B_z}{\partial z}\right)$$

$$F_z = \int_V f_z dv \qquad M_z = \int_V J_z dv$$

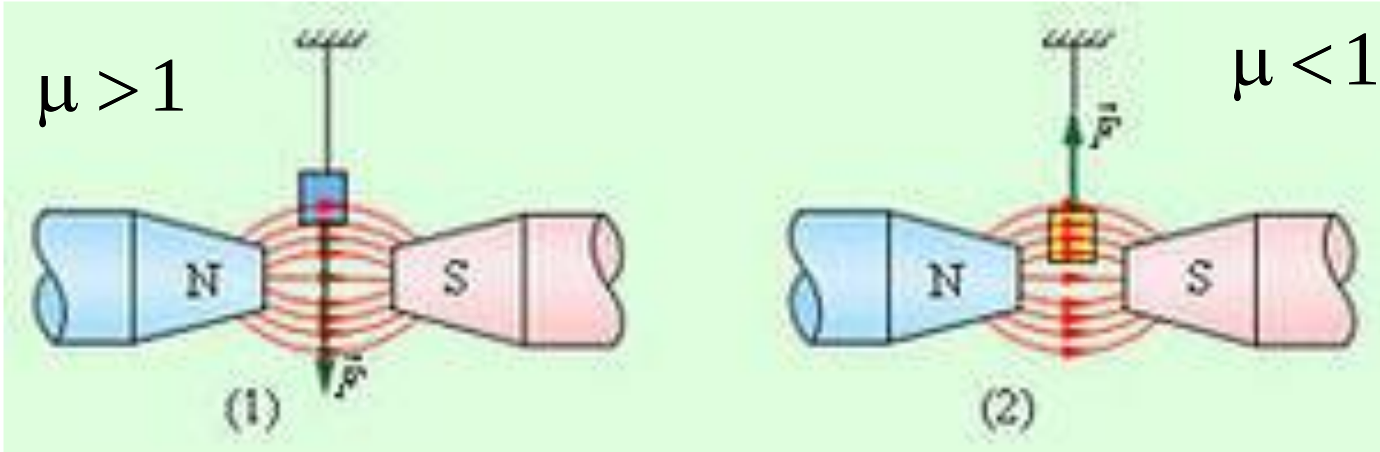
$$\int_V f_z dv = \int_V J_x\left(\frac{\partial B_x}{\partial z}\right)dv + \int_V J_y\left(\frac{\partial B_y}{\partial z}\right)dv + \int_V J_z\left(\frac{\partial B_z}{\partial z}\right)dv$$

$$f_z = J_x\left(\frac{\partial B_x}{\partial z}\right) + J_y\left(\frac{\partial B_y}{\partial z}\right) + J_z\left(\frac{\partial B_z}{\partial z}\right)$$

$$\mathbf{J} = (\mu - 1)\mathbf{B} / \mu_0\mu \qquad \mathbf{H} = \mathbf{B} / \mu\mu_0 = \mathbf{B} / \mu_0 - \mathbf{J}$$

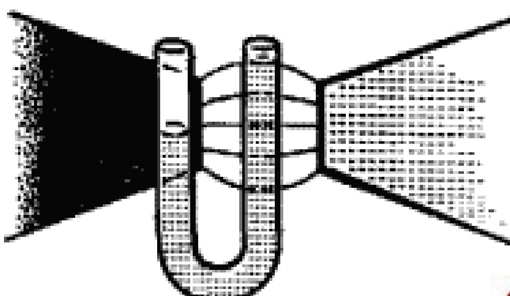
$$f_z = \frac{\mu - 1}{\mu_0\mu}\left(B_x\left(\frac{\partial B_x}{\partial z}\right) + B_y\left(\frac{\partial B_y}{\partial z}\right) + B_z\left(\frac{\partial B_z}{\partial z}\right)\right) = \frac{\mu - 1}{2\mu_0\mu} \cdot \frac{\partial}{\partial z}\mathbf{B}^2$$

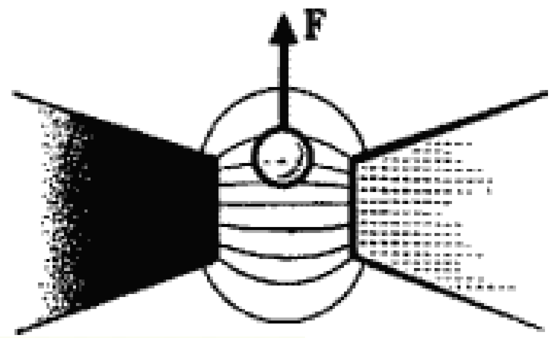
$$\mathbf{f} = \frac{\mu - 1}{2\mu_0\mu} \cdot \mathrm{grad}\left(\mathbf{B}^2\right)$$



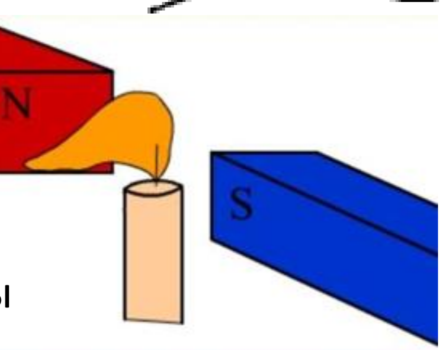
$$\mathbf{f} = \frac{\mu - 1}{2\mu_0\mu} \cdot \operatorname{grad}\left(\mathbf{B}^2\right)$$

$\mu > 1$





$\mu < 1$



Продукты сгорания –
диамагнитные частицы

Момент силы действующей на элементарный ток

$$\mathbf{N} = \int_V [\mathbf{r}\times[\mathbf{j}\times\mathbf{B}]]dv \qquad \mathbf{B} = \mathbf{B}(0) = \mathbf{B}_0$$

$$\mathbf{N} = \int_V [\mathbf{r}\times[\mathbf{j}\times\mathbf{B}]]dv = \int_V \mathbf{j}(\mathbf{r}\mathbf{B}_0)dv - \mathbf{B}_0\int_V \mathbf{j}\mathbf{r}dv$$

$$=0$$

$$\left(\int_V j_x x dv = \int_V j_y y dv = \int_V j_z z dv = 0\right)$$

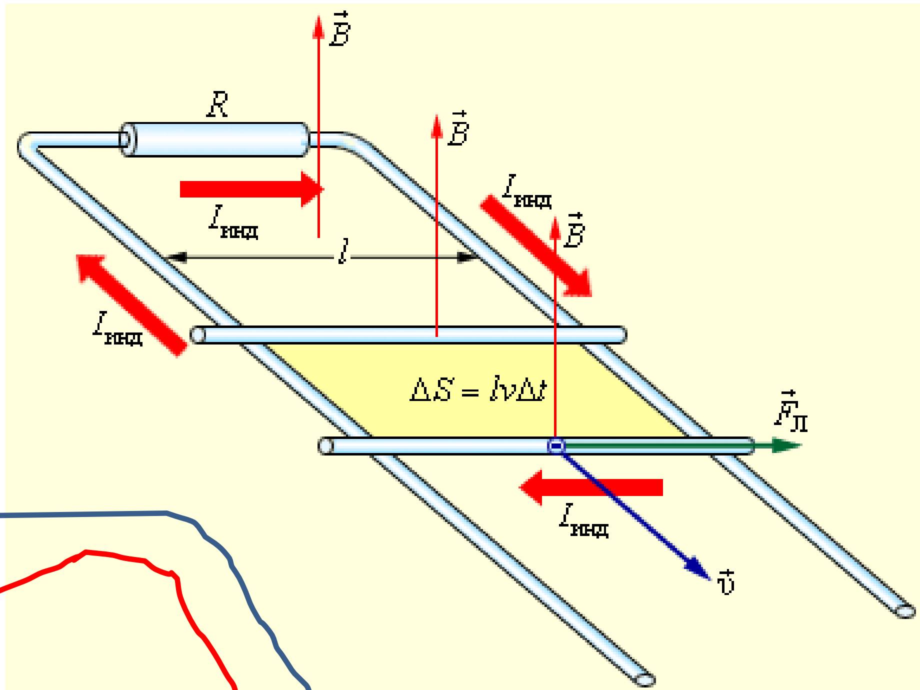
$$=0$$

$$N_x = \int_V j_x(xB_{0x} + yB_{0y} + zB_{0z})dv = B_{0x}\int_V j_x x dv +$$

$$+B_{0y}\int_V j_x y dv + B_{0z}\int_V j_x z dv = -B_{0y}M_z + B_{0z}M_y$$

$$\mathbf{N} = [\mathbf{M}\times\mathbf{B}]$$

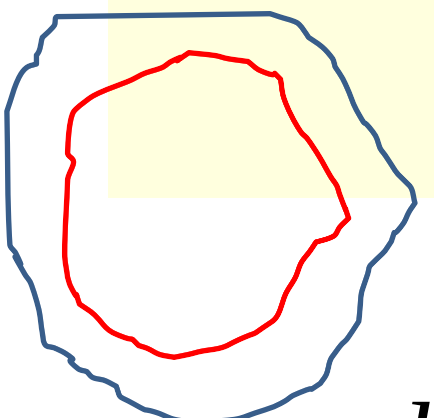
Электрическое поле в проводнике движущемся в стационарном магнитном поле



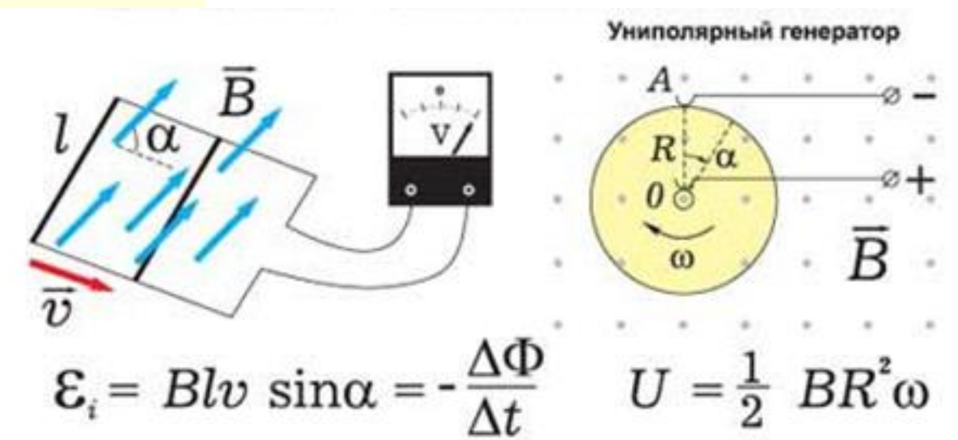
$$\mathbf{F} = q[\mathbf{v} \times \mathbf{B}]$$

$$\mathbf{E} = [\mathbf{v} \times \mathbf{B}]$$

$$\varepsilon_{инд} = vBL$$



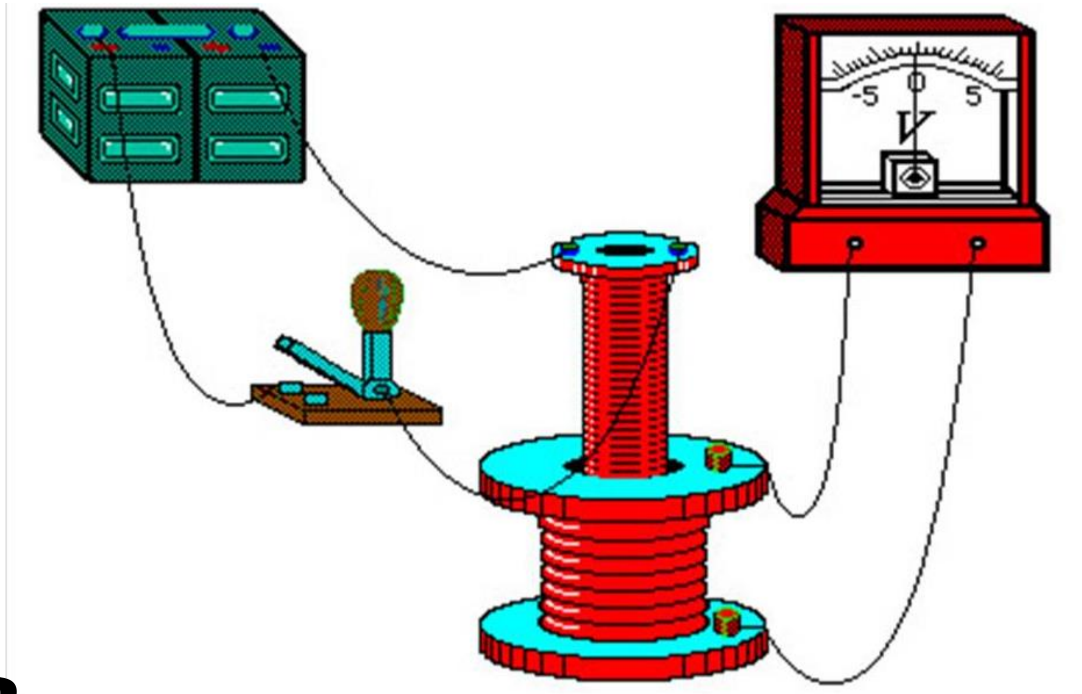
$$\varepsilon_{инд} = -\frac{d\Phi}{dt}$$

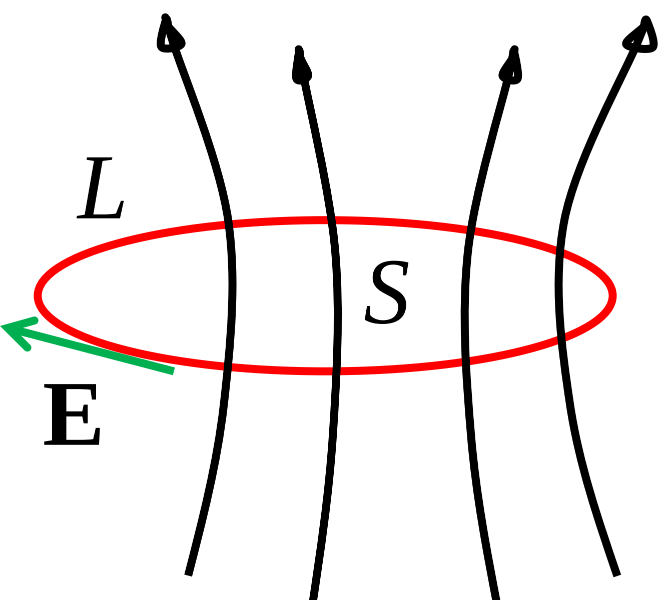


Закон электромагнитной индукции Фарадея

$$\varepsilon_{\text{инд}} = -\frac{d\Phi}{\partial t}$$

$$\oint_L \mathbf{E}\,d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t}\cdot d\mathbf{S}$$



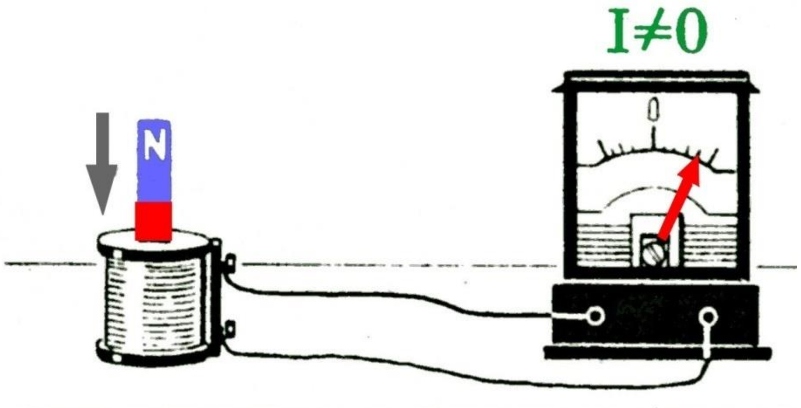


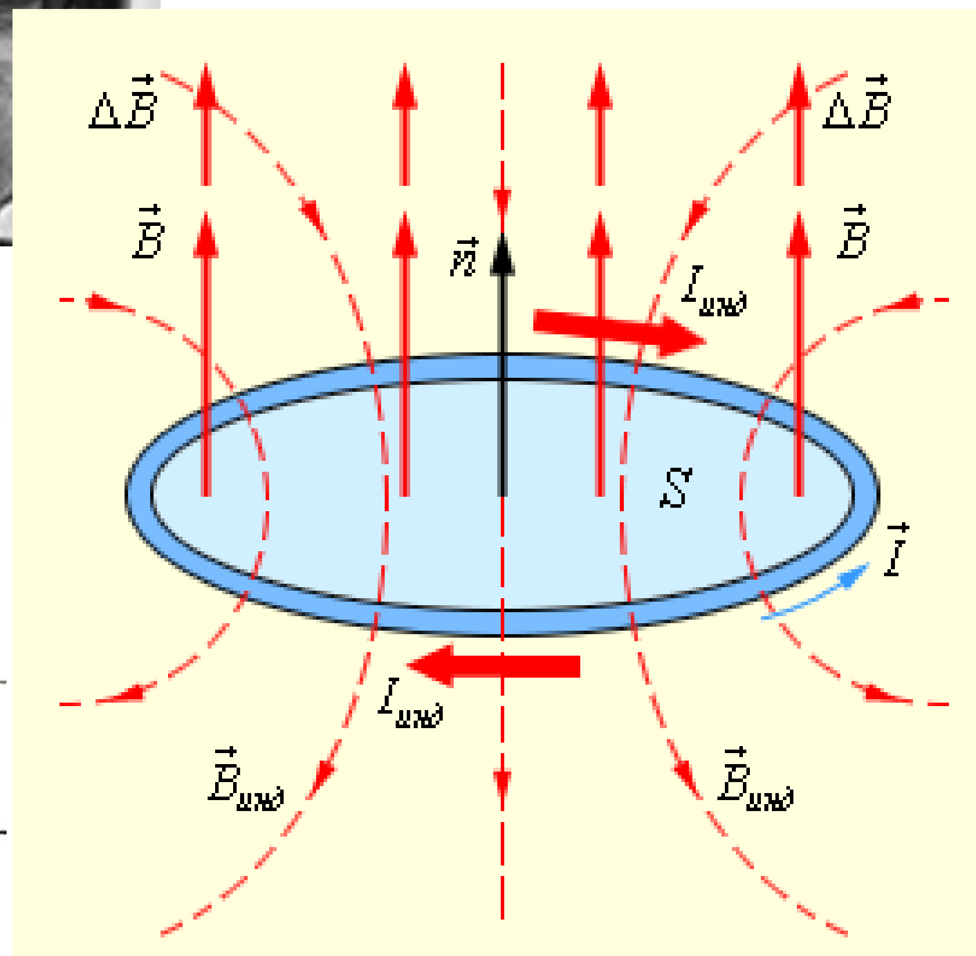
$$\frac{\partial \mathbf{B}}{\partial t} > 0$$

$$\operatorname{rot}\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\oint_L \mathbf{E}d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}$$

$$\mathrm{rot}\,\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$



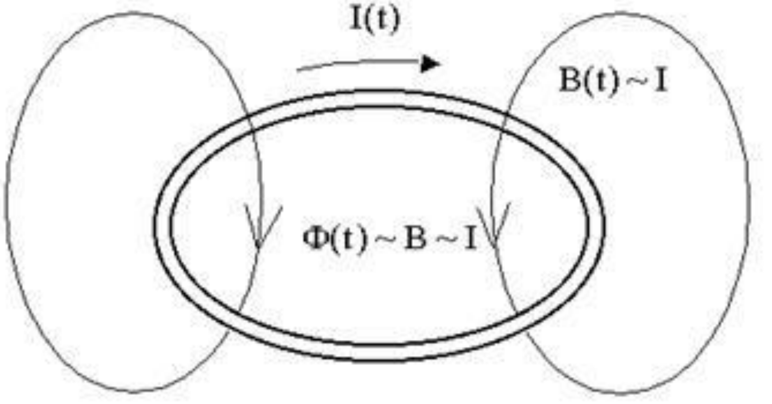


$$0 = \operatorname{div}\operatorname{rot}\mathbf{E} = -\operatorname{div}\frac{\partial \mathbf{B}}{\partial t} = -\frac{\partial}{\partial t}\operatorname{div}\mathbf{B}$$

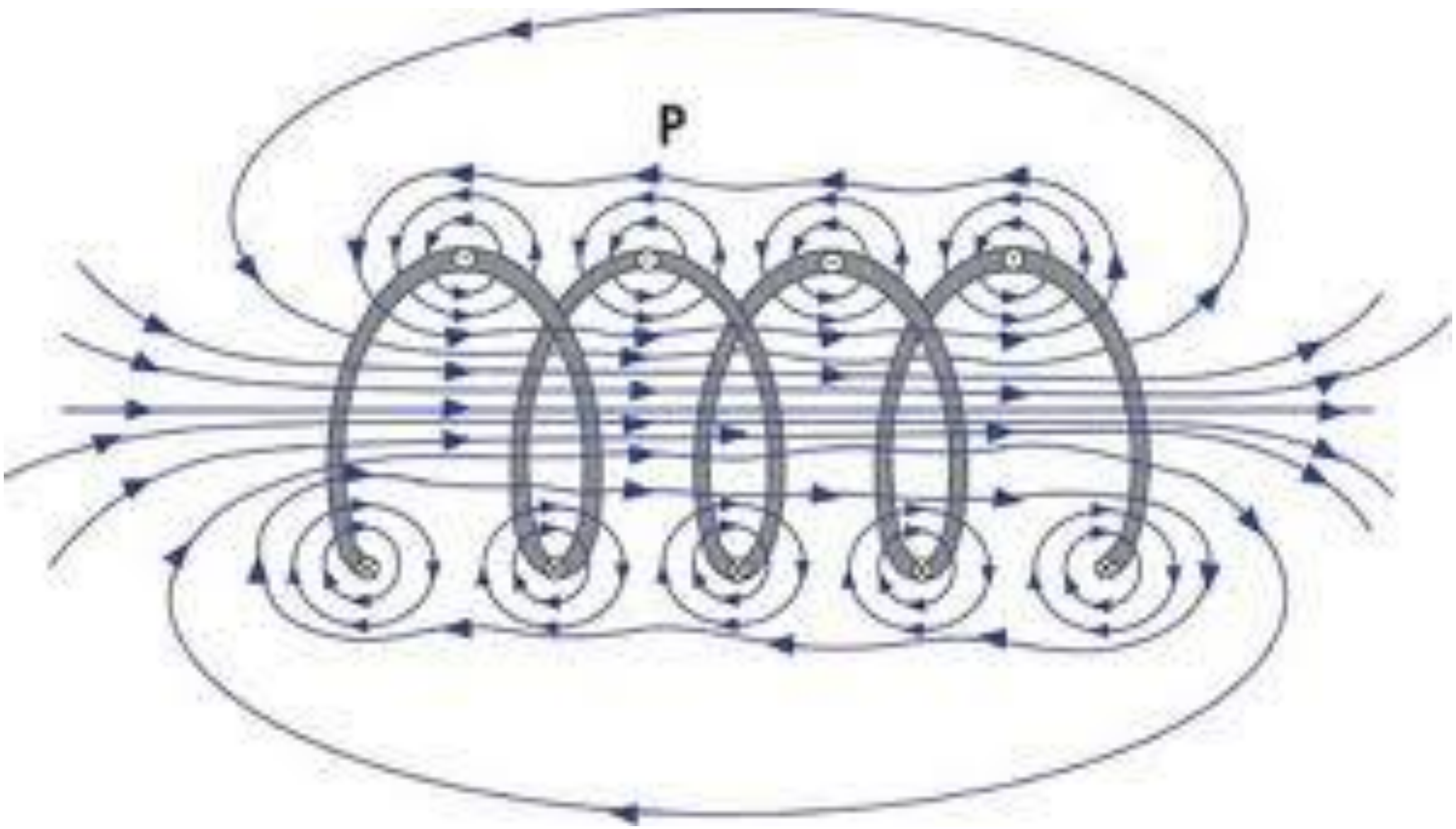
$$\operatorname{div}\mathbf{B} = \text{const} \quad \Rightarrow \quad \operatorname{div}\mathbf{B} = 0$$

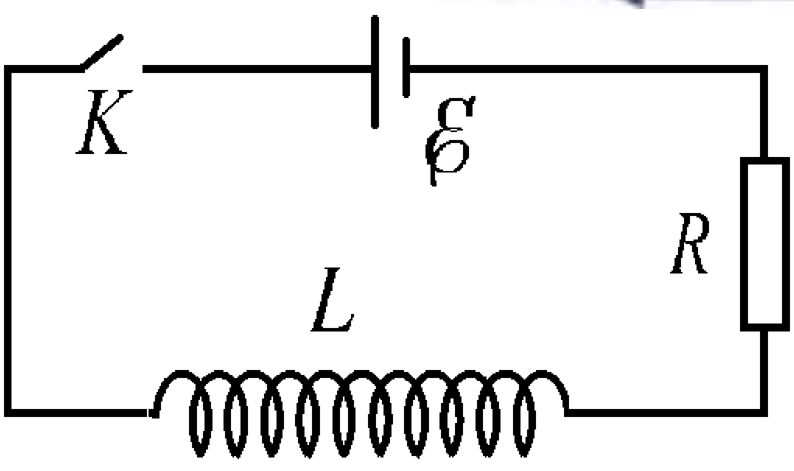
Явление самоиндукции

Джозеф Генри



$$\varepsilon_{\text{инд}} = -L\frac{dI}{dt}$$

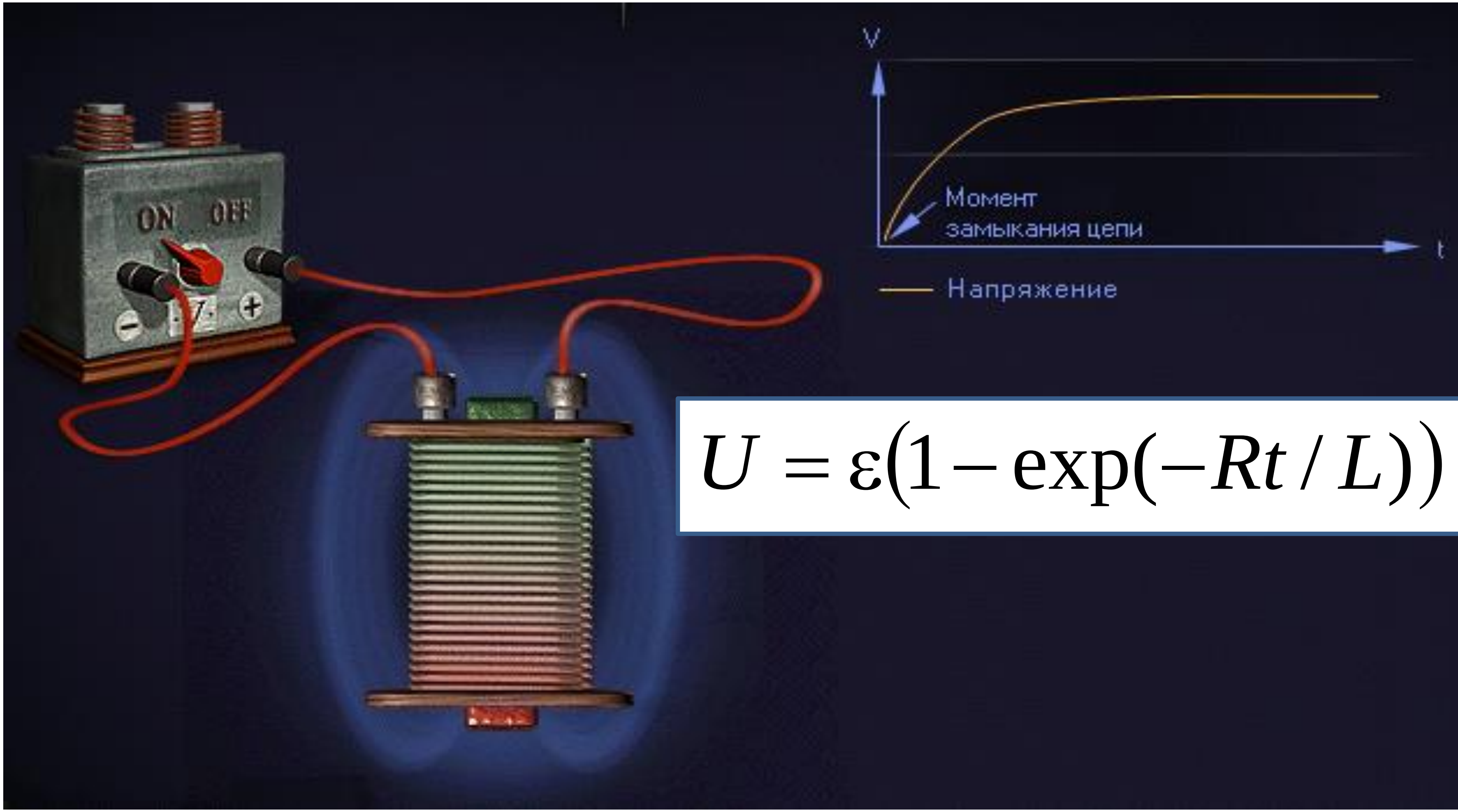




$$\varepsilon - L\frac{dI}{dt} = IR$$

$$I(0) = 0$$

$$I = \frac{\varepsilon}{R}\left(1 - \exp(-Rt/L)\right)$$



$$A(T) = \int_0^T I\varepsilon dt = \frac{\varepsilon^2}{R}\int_0^T \left(1 - \exp(-Rt / L)\right)dt =$$

$$= \frac{\varepsilon^2 T}{R} - \frac{\varepsilon^2 L}{R^2}\left(1 - \exp(-RT / L)\right)$$

$$Q(T)=\int_0^T I^2(t)Rdt=$$

$$I=\frac{\varepsilon}{R}\left(1-\exp(-Rt/L)\right)$$

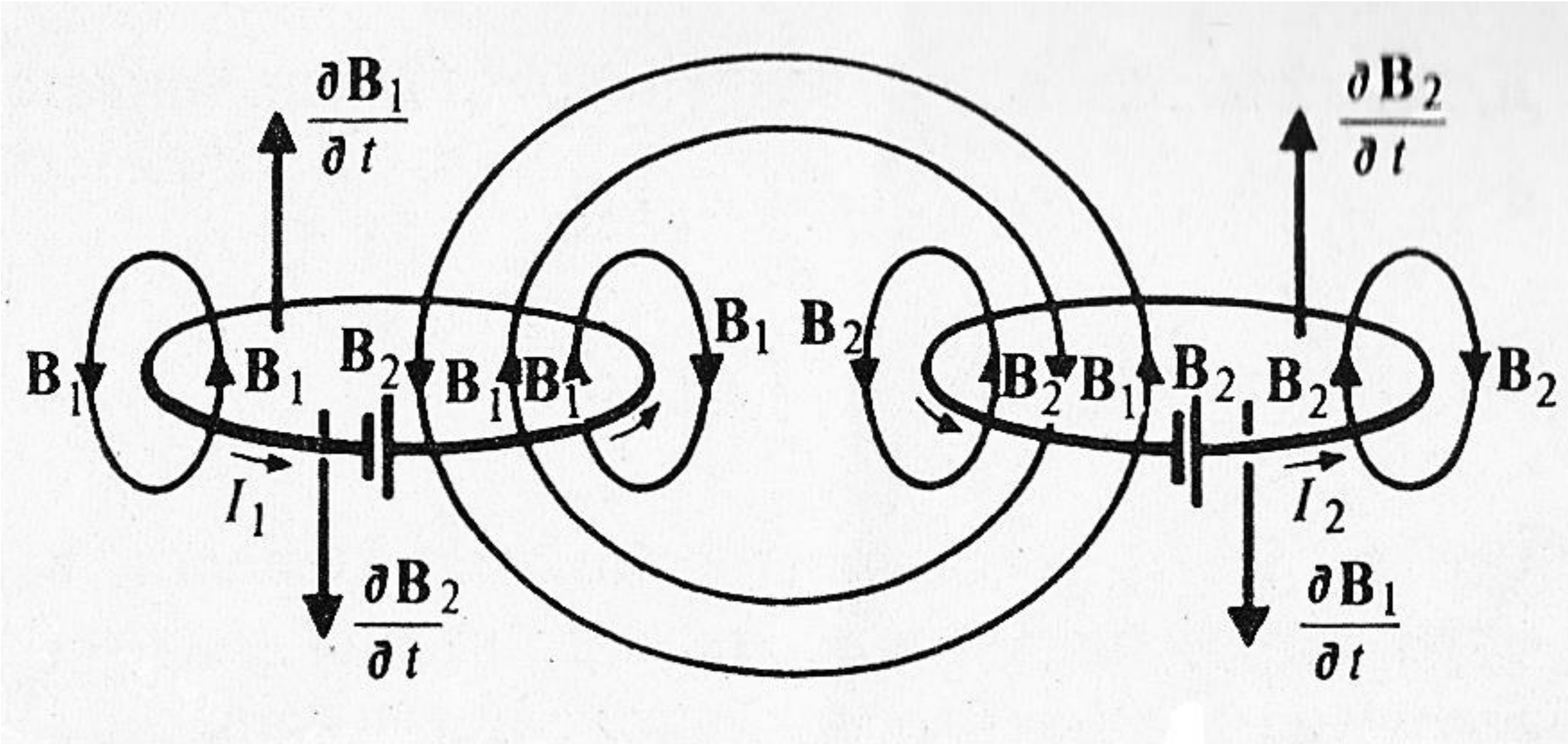
$$=\frac{\varepsilon^2}{R}\int_0^T\left(1-2\exp(-Rt/L)+\exp(-2Rt/L)\right)dt$$

$$=\frac{\varepsilon^2 T}{R}-\frac{2L\varepsilon^2}{R^2}\left(1-\exp(-RT/L)\right)+$$

$$+\frac{L\varepsilon^2}{2R^2}\left(1-\exp(-2RT/L)\right)$$

$$W(T)=A-Q=\frac{\varepsilon^2 L}{R^2}\left(1-\exp(-RT/L)\right)-\frac{L\varepsilon^2}{2R^2}\left(1-\exp(-2RT/L)\right)=$$

$$=\frac{L\varepsilon^2}{2R^2}[1-2\exp(-RT/L)+\exp(-2RT/L)]=\frac{LI^2(T)}{2}$$



$$\varepsilon_1 - \frac{d\Phi_{11}}{dt} - \frac{d\Phi_{12}}{dt} = \varepsilon_1 - L_{11}\frac{dI_1}{dt} - L_{12}\frac{dI_2}{dt} = I_1 R_1$$

$$\varepsilon_2 - \frac{d\Phi_{21}}{dt} - \frac{d\Phi_{22}}{dt} = \varepsilon_2 - L_{21}\frac{dI_1}{dt} - L_{22}\frac{dI_2}{dt} = I_2 R_2$$

Энергия магнитного поля

$$W = \frac{1}{2}\sum_{\substack{i=1\\ j=1}}^{N} L_{ij} I_i I_j$$

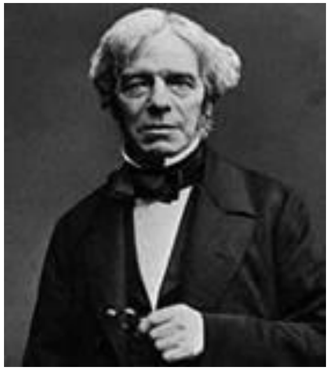
Докажем, что $L_{ij} = L_{ji}$

$$\Phi_{21} = \int_{S_2} \mathbf{B}_1 d\mathbf{S}_2 = \int_{S_2} \mathrm{rot}\mathbf{A}_1 d\mathbf{S}_2 = \int_{\tilde{L}_2} \mathbf{A}_1 d\mathbf{l}_2 = \frac{\mu\mu_0 I_1}{4\pi}\int_{\tilde{L}_2}\int_{\tilde{L}_1}\frac{d\mathbf{l}_1 d\mathbf{l}_2}{r_{12}} = L_{21} I_1$$

$$L_{21} = \frac{\mu\mu_0}{4\pi}\int_{\tilde{L}_2}\int_{\tilde{L}_1}\frac{d\mathbf{l}_1 d\mathbf{l}_2}{r_{12}} = \frac{\mu\mu_0}{4\pi}\int_{\tilde{L}_1}\int_{\tilde{L}_2}\frac{d\mathbf{l}_2 d\mathbf{l}_1}{r_{21}} = L_{12}$$

$$W = \frac{1}{2}\sum_{\substack{i=1\\ j=1}}^{N} L_{ij} I_i I_j \quad \Longrightarrow \quad W = \int_V w\,dv \qquad w = \frac{\mathbf{BH}}{2}$$

Нестационарное электромагнитное поле в вакууме



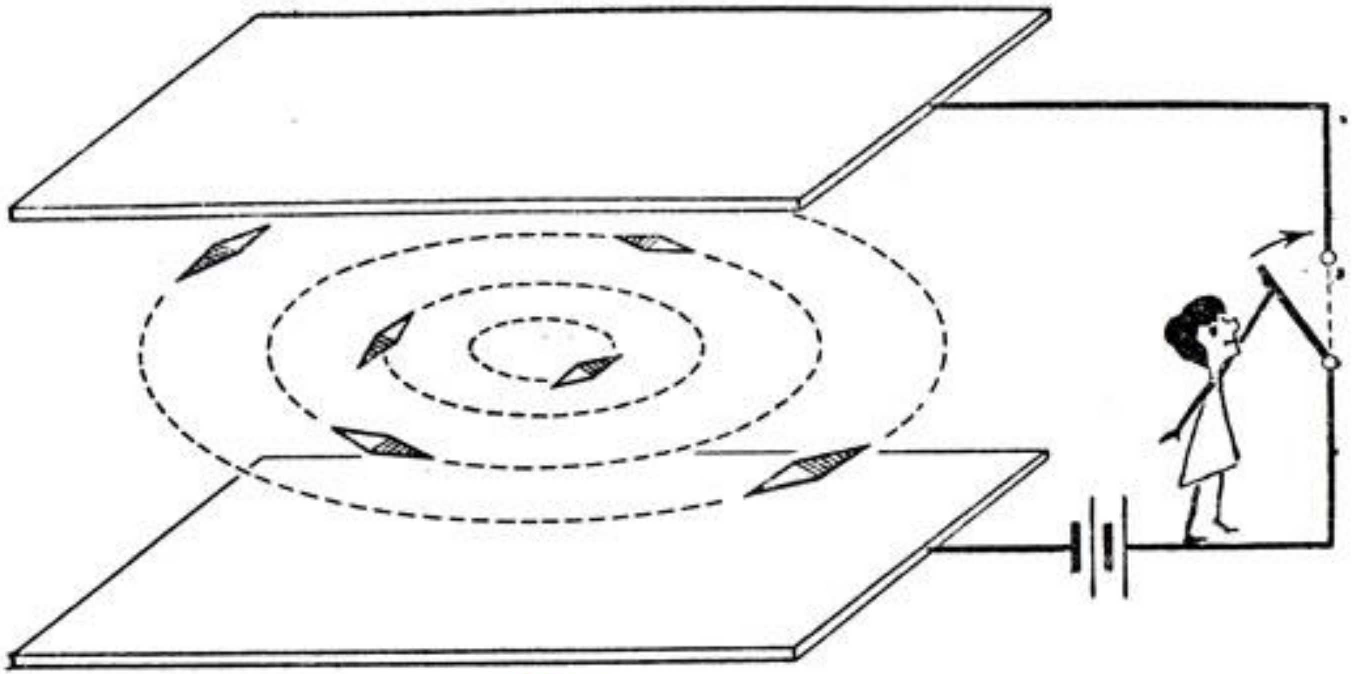
$$\oint_L \mathbf{E}d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{dt} \cdot d\mathbf{S} \Rightarrow \operatorname{rot}\mathbf{E} = -\frac{\partial \mathbf{B}}{dt}$$

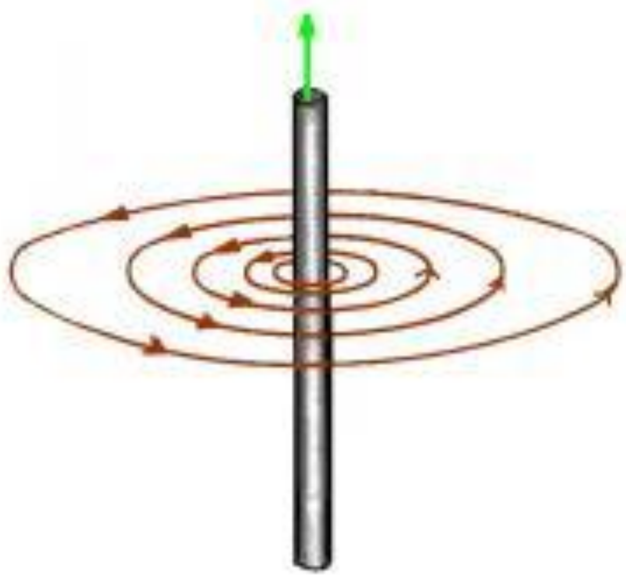
$$\int_S \mathbf{B} \cdot d\mathbf{S} = 0 \Rightarrow \operatorname{div}\mathbf{B} = 0$$

$$\oint_L \mathbf{B}d\mathbf{l} = \mu_0 \int_S \mathbf{j} \cdot d\mathbf{S} + \mu_0\varepsilon_0 \int_S \frac{\partial \mathbf{E}}{\partial t} \cdot d\mathbf{S}$$

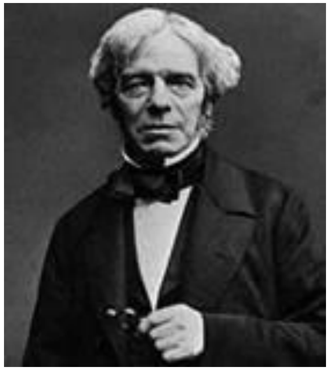
$$\Downarrow$$

$$\operatorname{rot}\mathbf{B} = \mu_0 \mathbf{j} + \mu_0\varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$





Нестационарное электромагнитное поле в вакууме



$$\oint_L \mathbf{E}d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S} \Rightarrow \operatorname{rot} \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\int_S \mathbf{B} \cdot d\mathbf{S} = 0 \Rightarrow \operatorname{div} \mathbf{B} = 0$$

$$\oint_L \mathbf{B}d\mathbf{l} = \mu_0 \int_S \mathbf{j} \cdot d\mathbf{S} + \mu_0 \varepsilon_0 \int_S \frac{\partial \mathbf{E}}{\partial t} \cdot d\mathbf{S}$$

$$\Downarrow$$

$$\operatorname{rot} \mathbf{B} = \mu_0 \mathbf{j} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\operatorname{rot}\mathbf{B} = \mu_0 \mathbf{j} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$0 = \operatorname{div}\operatorname{rot}\mathbf{B}(\mathbf{r},t) = \operatorname{div}\mathbf{j} + \varepsilon_0 \operatorname{div}\frac{\partial \mathbf{E}(\mathbf{r},t)}{\partial t} = -\frac{\partial \rho}{\partial t} + \varepsilon_0 \operatorname{div}\frac{\partial \mathbf{E}}{\partial t} =$$

$$= \frac{\partial}{\partial t}\left(-\rho + \varepsilon_0 \operatorname{div}\mathbf{E}\right) \Rightarrow \operatorname{div}\mathbf{E}(\mathbf{r},t) = \rho/\varepsilon_0 + f(\mathbf{r})$$

$$f(\mathbf{r}) = 0 \qquad \operatorname{div}\mathbf{E} = \rho/\varepsilon_0 \Rightarrow \int_S \mathbf{E}\cdot d\mathbf{S} = \frac{1}{\varepsilon_0}\int_V \rho\, dv$$

$$\operatorname{rot}\mathbf{B} = \mu_0 \mathbf{j} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\operatorname{div}\mathbf{B} = 0$$

$$\operatorname{rot}\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\operatorname{div}\mathbf{E} = \rho/\varepsilon_0$$

1.Восемь уравнений, но шесть неизвестных (второе и четвертое уравнения получаются из первого и третьего с помощью уравнения непрерывности.

2.Принцип суперпозиции электромагнитных полей.

3.Для решения нужны начальные и граничные условия

Уравнения Максвелла в среде

$$\operatorname{rot}\mathbf{B}_m = \mu_0\overline{\rho\mathbf{v}} + \mu_0\varepsilon_0\frac{\partial\mathbf{E}_m}{\partial t} \Rightarrow \mathbf{?}$$

$$\operatorname{div}\mathbf{B}_m = 0 \Rightarrow \operatorname{div}\mathbf{B} = 0$$

$$\operatorname{rot}\mathbf{E}_m = -\frac{\partial\mathbf{B}_m}{\partial t} \Rightarrow \operatorname{rot}\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$$

$$\operatorname{div}\mathbf{E}_m = \overline{\rho}/\varepsilon_0 \Rightarrow \operatorname{div}\mathbf{D} = \rho_{своб}$$

$$\overline{f} = \frac{1}{2\tau}\int_{-\tau}^{\tau}\frac{1}{v_0}\int_{v_0} f_m\,dv\,dt$$

$$\overline{\rho} = \rho_{своб} + \rho_{связ} = \rho_{своб} - \operatorname{div}\mathbf{P}$$

$$\overline{\rho\mathbf{v}} = \mathbf{j}_{пров} + \mathbf{j}_{мол} + \gamma\partial\mathbf{E}/\partial t = \mathbf{j} + \operatorname{rot}\mathbf{J} + \gamma\partial\mathbf{E}/\partial t \qquad (\mathbf{j} \propto \mathbf{B})$$

$$0 = \frac{\partial(\rho_{своб} + \rho_{связ})}{\partial t} + \mathrm{div}(\mathbf{j}_{пров} + \mathbf{j}_{мол} + \gamma \partial \mathbf{E} / \partial t) =$$

$$= \frac{\partial \rho_{своб}}{\partial t} + \mathrm{div}\, \mathbf{j}_{пров} + \frac{\partial(-\mathrm{div}\, \mathbf{P})}{\partial t} + \mathrm{div}\,\mathrm{rot}\, \mathbf{J} + \mathrm{div}\, \gamma \partial \mathbf{E} / \partial t =$$

$$= 0$$

$$= \frac{\partial\, \mathrm{div}(-\mathbf{P} + \gamma \mathbf{E})}{\partial t} \Rightarrow \mathbf{P} = \gamma \mathbf{E}$$

$$\mathrm{rot}\, \mathbf{B}_m = \mu_0 \overline{\rho \mathbf{v}} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}_m}{\partial t}$$

$$\mathrm{rot}\, \mathbf{B}_m = \mu_0 (\mathbf{j} + \mathrm{rot}\, \mathbf{J} + \partial \mathbf{P} / \partial t) + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}_m}{\partial t}$$

$$\mathbf{H} = \mathbf{B} / \mu_0 - \mathbf{J} \qquad \mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$$

$$\mathrm{rot}\, \mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{P}}{\partial t} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t}$$

Уравнения Максвелла в среде

$$\mathrm{rot}\,\mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t} \qquad \oint_L \mathbf{H}d\mathbf{l} = \int_S \mathbf{j}d\mathbf{S} + \int_S \frac{\partial \mathbf{D}}{\partial t}d\mathbf{S}$$

$$\mathrm{div}\,\mathbf{B} = 0 \qquad \oint_S \mathbf{B}d\mathbf{S} = 0$$

$$\mathrm{rot}\,\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \oint_L \mathbf{E}d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t}d\mathbf{S}$$

$$\mathrm{div}\,\mathbf{D} = \rho \qquad \oint_S \mathbf{D}d\mathbf{S} = \int_V \rho dv$$

Материальные уравнения

$$\mathbf{D} = \varepsilon\varepsilon_0\mathbf{E} \qquad \mathbf{B} = \mu\mu_0\mathbf{H}$$

Уравнения Максвелла в среде

$$\mathrm{rot}\,\mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t}$$

$$\mathrm{div}\,\mathbf{B} = 0$$

$$\mathrm{rot}\,\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\mathbf{div}\,\mathbf{D} = \rho$$

$$\mathbf{D} = \varepsilon\varepsilon_0\mathbf{E}$$

$$\mathbf{B} = \mu\mu_0\mathbf{H}$$

1. Заряженные тела и провода с токами неподвижны.
2. Материальные константы не зависят от времени.
3. Отсутствуют постоянные магниты и ферромагнетики.
4. При заданные граничных и начальных условиях ее решение единственно.

Граничные условия

$$\mathbf{D}_{n2} - \mathbf{D}_{n1} = 0 \qquad \mathbf{B}_{n2} - \mathbf{B}_{n1} = 0$$

$$\mathbf{E}_{t2} - \mathbf{E}_{t1} = 0 \qquad \mathbf{H}_{t2} - \mathbf{H}_{t1} = \mathbf{i}$$

Единственность решения

$$\mathbf{D}_2 - \mathbf{D}_1 = \mathbf{D} \qquad \mathbf{B}_2 - \mathbf{B}_1 = \mathbf{B} \qquad \mathbf{E}_2 - \mathbf{E}_1 = \mathbf{E} \qquad \mathbf{H}_2 - \mathbf{H}_1 = \mathbf{H}$$

$$\mathrm{rot}\,\mathbf{H} = \sigma\mathbf{E} + \frac{\partial \mathbf{D}}{\partial t}, \quad \mathrm{div}\,\mathbf{B} = 0, \quad \mathrm{rot}\,\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \ \mathrm{div}\,\mathbf{D} = 0$$

В начальный момент времени $\mathbf{D} = \mathbf{E} = \mathbf{B} = \mathbf{H} = \mathbf{0}$

$\mathbf{D}$, $\mathbf{E}$, $\mathbf{B}$, $\mathbf{H}$ на границе равны нулю

$$\mathbf{a}\,\mathrm{rot}\,\mathbf{b} = \mathbf{b}\,\mathrm{rot}\,\mathbf{a} - \mathrm{div}[\mathbf{a}\times\mathbf{b}] \qquad \mathbf{b}=\mathbf{H} \qquad \mathbf{a}=\mathbf{E}$$

$$\int_V \sigma\mathbf{E}^2 dv = \int_V \mathbf{E}\left(\mathrm{rot}\,\mathbf{H} - \frac{\partial\mathbf{D}}{\partial t}\right)dV = \int_V \mathbf{H}\,\mathrm{rot}\,\mathbf{E}\,dv - \int_V \mathrm{div}[\mathbf{E}\times\mathbf{H}]dv -$$

$$-\varepsilon\varepsilon_0\int_V \mathbf{E}\frac{\partial\mathbf{E}}{\partial t}dv = -(\mu\mu_0)^{-1}\int_V \mathbf{B}\frac{\partial\mathbf{B}}{\partial t}dv - -\int_S [\mathbf{E}\times\mathbf{H}]d\mathbf{S} - \varepsilon\varepsilon_0\int_V \mathbf{E}\frac{\partial\mathbf{E}}{\partial t}dv =$$

$$= 0$$

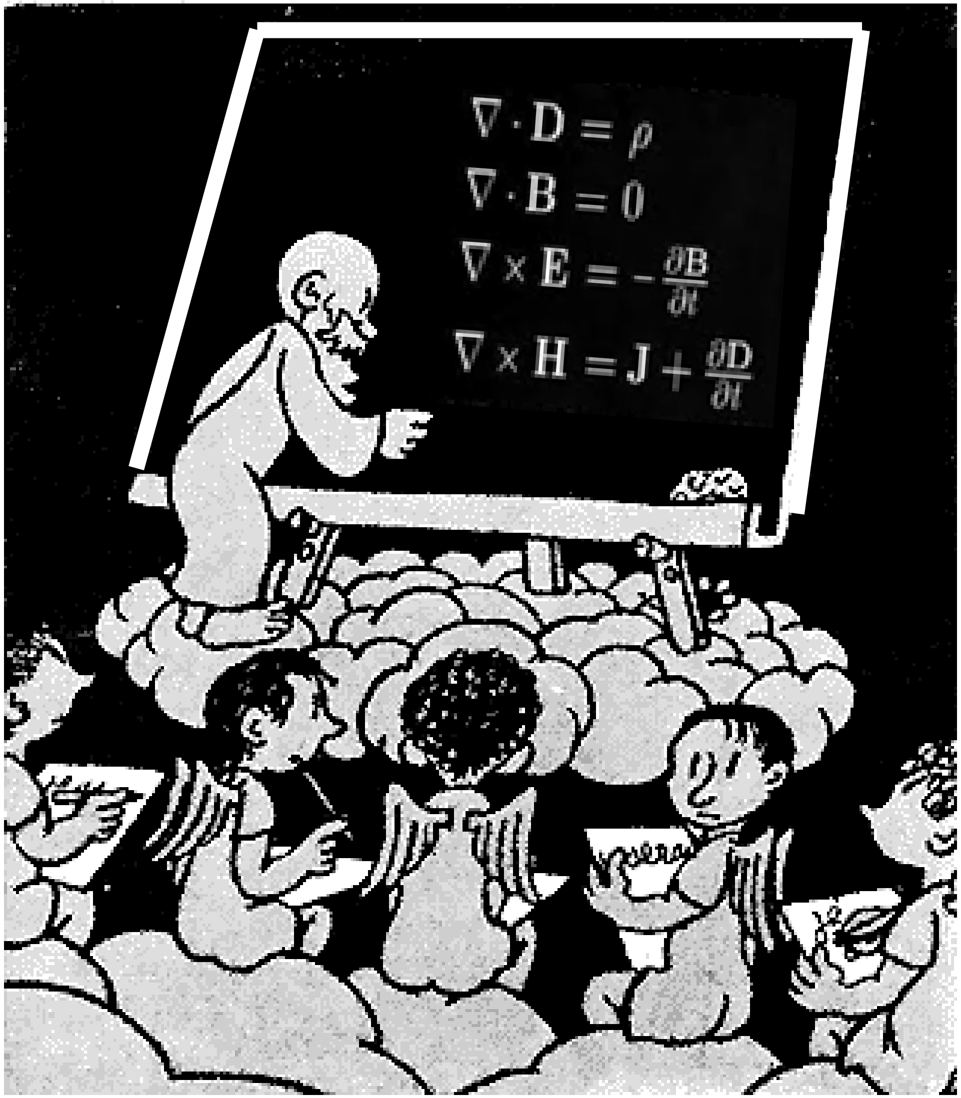
Нулевые граничные условия

$$= -\frac{\partial}{\partial t}\left(\int_V\left(\frac{1}{2}\varepsilon\varepsilon_0\mathbf{E}^2 + \frac{1}{2\mu\mu_0}\mathbf{B}^2\right)\right)dv$$

$$\frac{\partial}{\partial t}\left(\int_V\left(\frac{1}{2}\varepsilon\varepsilon_0\mathbf{E}^2 + \frac{1}{2\mu\mu_0}\mathbf{B}^2\right)dv\right) = -\int_V \sigma\mathbf{E}^2 dv \le 0$$

С ростом t $\int_V\left(\frac{1}{2}\varepsilon\varepsilon_0\mathbf{E}^2 + \frac{1}{2\mu\mu_0}\mathbf{B}^2\right)dv$ уменьшается, но из-за нулевых начальных условий $\left(\int_V\left(\frac{1}{2}\varepsilon\varepsilon_0\mathbf{E}^2 + \frac{1}{2}\mu\mu_0\mathbf{B}^2\right)dV\right)_{t=0} = 0$

$$\Rightarrow \quad \mathbf{D}=0,\quad \mathbf{E}=0,\quad \mathbf{B}=0,\quad \mathbf{H}=0$$



AND GOD SAID

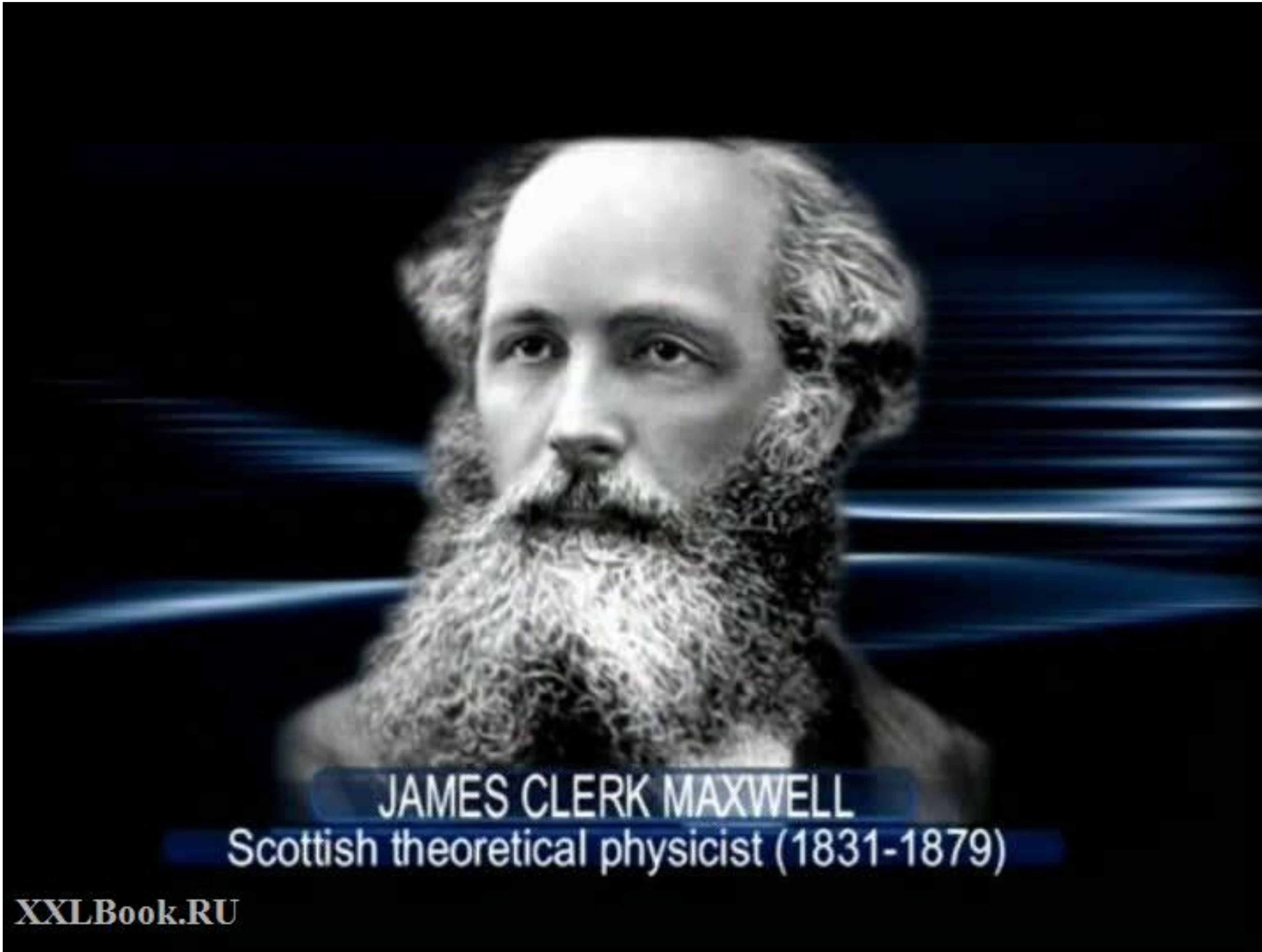
$$\operatorname{div} \mathbf{D} = \rho$$

$$\operatorname{div} \mathbf{B} = 0$$

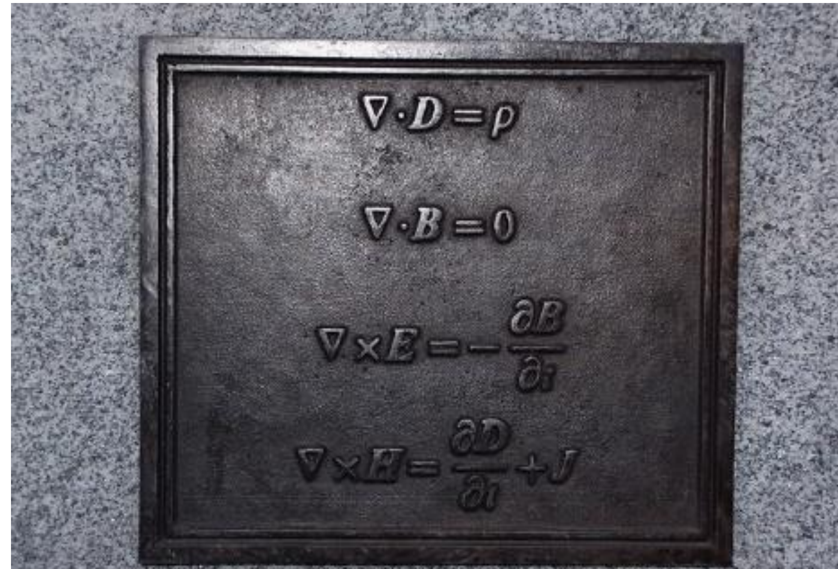
$$\operatorname{rot} \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\operatorname{rot} \mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t}$$

AND THEN THERE WAS LIGHT


JAMES CLERK MAXWELL
Scottish theoretical physicist (1831-1879)
XXLBook.RU


$\nabla \cdot D = \rho$
$\nabla \cdot B = 0$
$\nabla \times E = -\frac{\partial B}{\partial t}$
$\nabla \times H = J + \frac{\partial D}{\partial t}$


$\nabla \cdot D = \rho$
$\nabla \cdot B = 0$
$\nabla \times E = -\frac{\partial B}{\partial t}$
$\nabla \times H = \frac{\partial D}{\partial t} + J$

Закон сохранения энергии

$$W(t) = \frac{1}{2}\oint (\mathbf{ED} + \mathbf{BH})dV$$

$$\int_V \sigma \mathbf{E}^2 dv = \int_V \mathbf{jE}dv = \int_V \mathbf{E}\left(\operatorname{rot}\mathbf{H} - \frac{\partial \mathbf{D}}{\partial t}\right)dV = \int_V \mathbf{H}\operatorname{rot}\mathbf{E}dv -$$

$$-\int_V \operatorname{div}[\mathbf{E}\times\mathbf{H}]dv - \int_V \mathbf{E}\frac{\partial \mathbf{D}}{\partial t}dv =$$

$$\mathbf{a}\operatorname{rot}\mathbf{b} = \mathbf{b}\operatorname{rot}\mathbf{a} - \operatorname{div}[\mathbf{a}\times\mathbf{b}]$$

$$\mathbf{a} = \mathbf{E} \qquad \mathbf{b} = \mathbf{H}$$

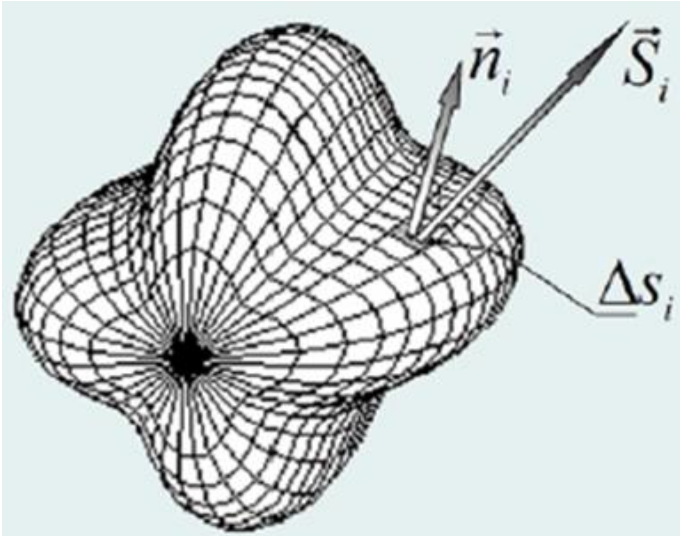
$$= -\int_V \mathbf{H}\frac{\partial \mathbf{B}}{\partial t}dv - \int_S [\mathbf{E}\times\mathbf{H}]d\mathbf{S} - \int_V \mathbf{E}\frac{\partial \mathbf{D}}{\partial t}dv \Rightarrow$$

$$\mathbf{H}\frac{\partial \mathbf{B}}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t}\mathbf{BH}$$

$$\Rightarrow \int_V \mathbf{j}\cdot\mathbf{E}dV = -\left[\frac{1}{2}\int_V\left(\frac{\partial}{\partial t}\mathbf{BH} + \frac{\partial}{\partial t}\mathbf{ED}\right)dV\right] - \int_S [\mathbf{E}\times\mathbf{H}]d\mathbf{S}$$

$$\int_V \mathbf{j}\cdot\mathbf{E}dV = -\frac{\partial}{\partial t}\left(\frac{1}{2}\int_V (\mathbf{BH} + \mathbf{ED})dV\right) - \int_S [\mathbf{E}\times\mathbf{H}]d\mathbf{S}$$

$$\boxed{\frac{\partial W}{\partial t} = -\int_V \mathbf{j}\cdot\mathbf{E}dV - \int_S [\mathbf{E}\times\mathbf{H}]d\mathbf{S} \qquad \mathbf{S}_p = [\mathbf{E}\times\mathbf{H}]}$$



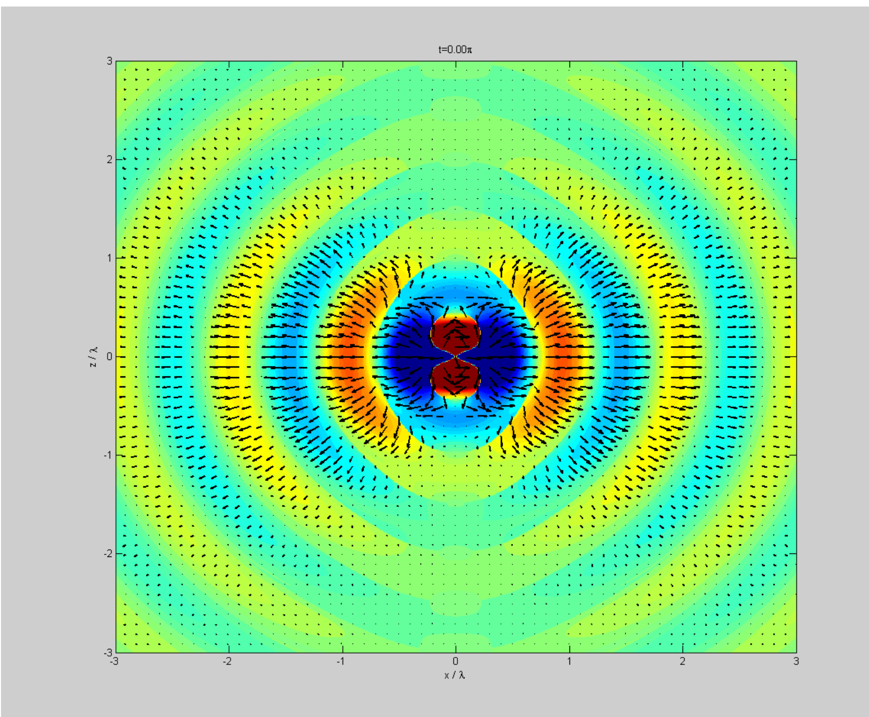
Диполь

$$\mathbf{S} = [\mathbf{E} \times \mathbf{H}]$$

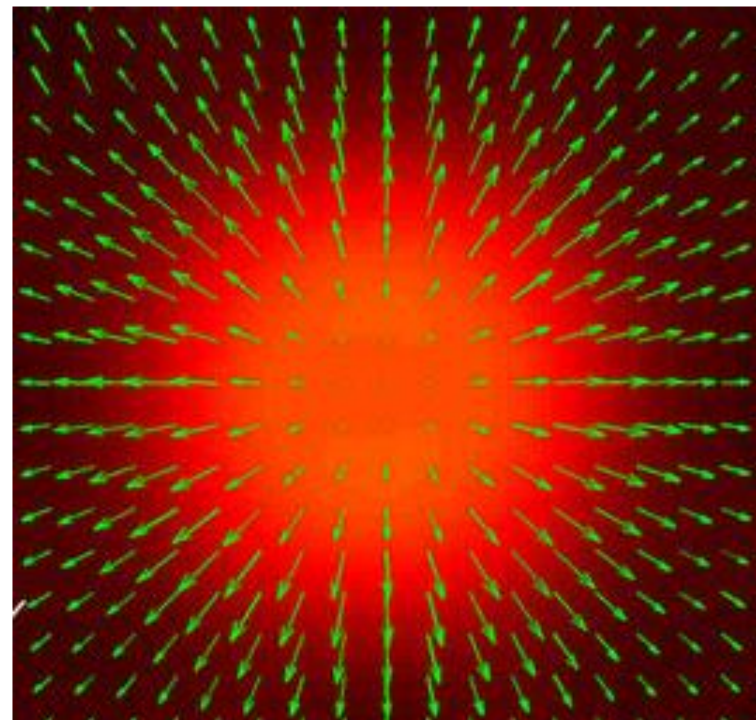
Отсутствие токов:

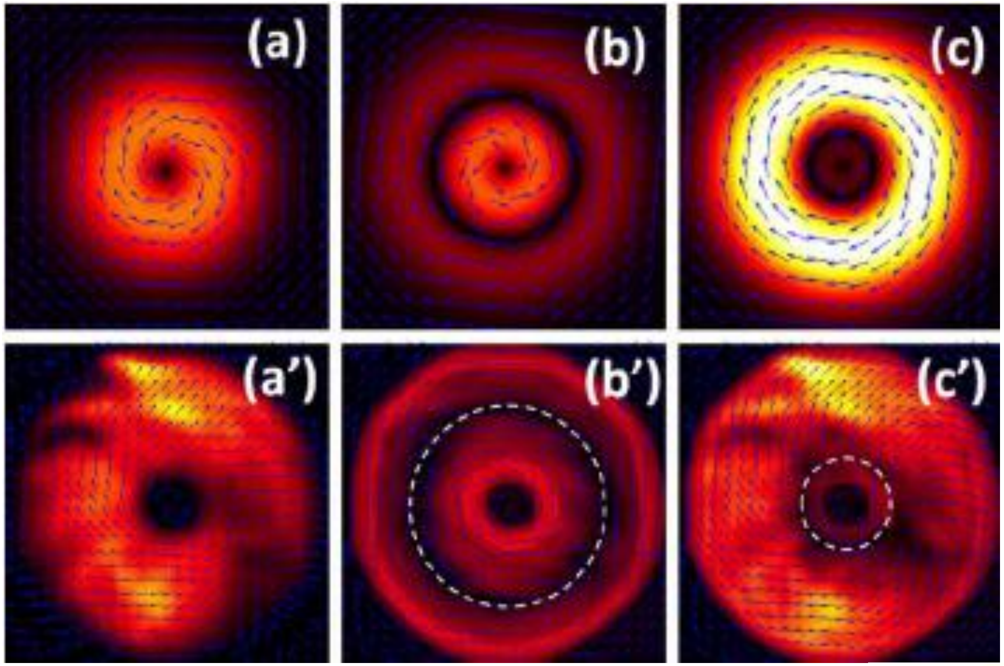
$$\frac{\partial}{\partial t}\left(\frac{1}{2}\int_V (\mathbf{BH} + \mathbf{ED})dV \right) + \int_S [\mathbf{E} \times \mathbf{H}]d\mathbf{S} = 0$$

Наночастица



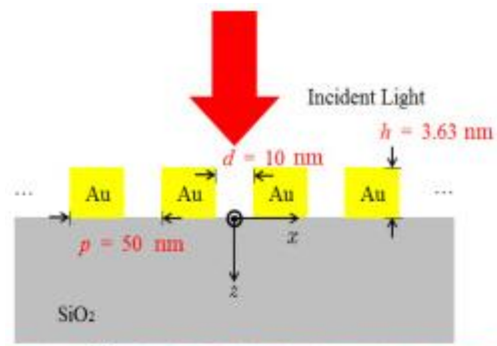
Finite-Difference Time-Domain method

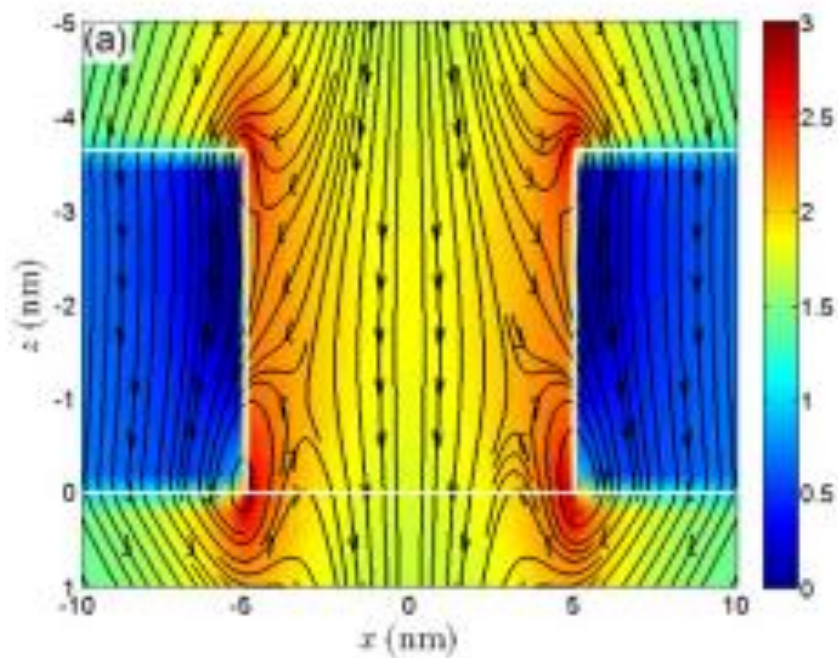




Лазерный пучок

Нанорешетка на поверхности





Существование электромагнитных волн

$$\operatorname{rot}\mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t} \qquad \operatorname{div}\mathbf{B} = 0$$

$$\operatorname{rot}\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \operatorname{div}\mathbf{D} = \rho = 0$$

$$= 0$$

$$\operatorname{rot}\operatorname{rot}\mathbf{E} = \operatorname{grad}\operatorname{div}\mathbf{E} - \underline{\Delta\mathbf{E}} = -\frac{\partial}{\partial t}\operatorname{rot}\mathbf{B} =$$

$$= -\frac{\partial}{\partial t}\mu\mu_0\frac{\partial \mathbf{D}}{\partial t} - \frac{\partial \mathbf{j}}{\partial t}\mu\mu_0 = -\varepsilon\varepsilon_0\mu\mu_0\underline{\frac{\partial^2 \mathbf{E}}{\partial t^2}} - \mu\mu_0\frac{\partial \mathbf{j}}{\partial t}$$

$$\Delta\mathbf{E} - \frac{1}{v^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = \mu\mu_0\frac{\partial \mathbf{j}}{\partial t} \qquad \varepsilon\varepsilon_0\mu\mu_0 = \frac{\varepsilon\mu}{c^2} = \frac{1}{v^2}$$

$$\frac{\partial^2 \mathbf{E}}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \quad \Longrightarrow \quad \mathbf{E} = \mathbf{F}_1(t - z/v) + \mathbf{F}_2(t + z/v)$$

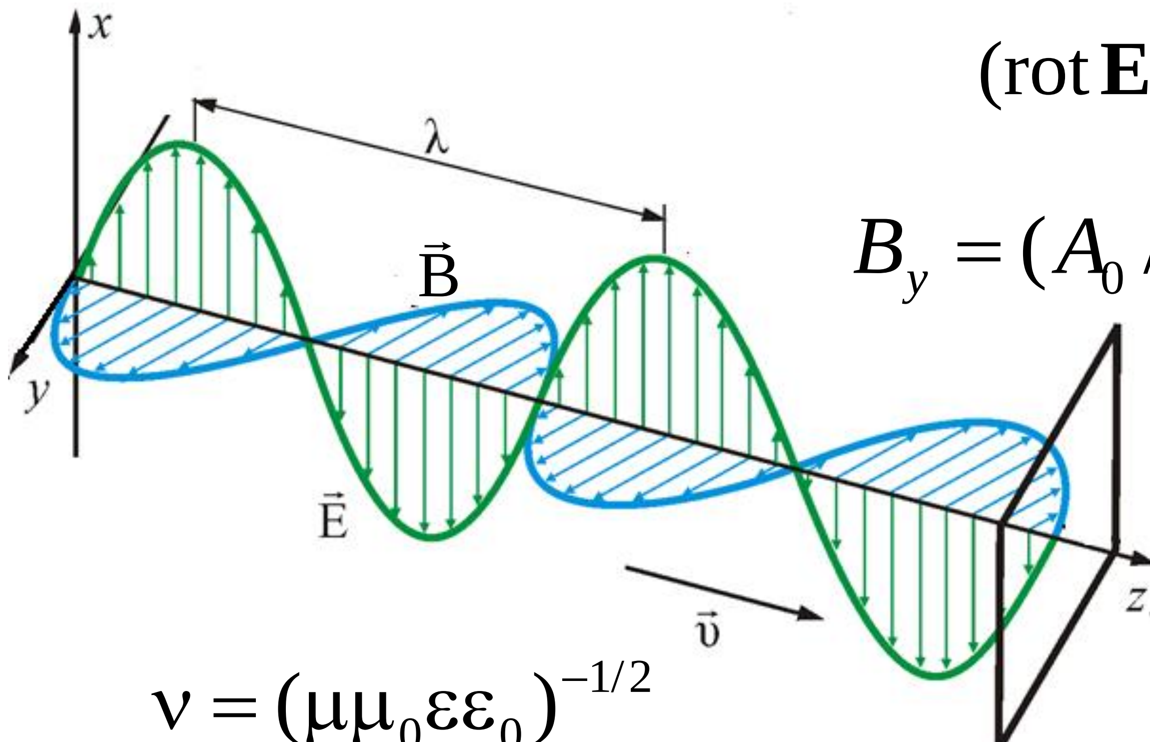
$$\mathbf{E} = \mathbf{A}_1 \sin[\omega(t - z/v) + \varphi_1] + \mathbf{A}_2 \sin[\omega(t + z/v) + \varphi_2]$$

$$E_x = A_0 \sin[\omega(t - z/v)]$$

$$\operatorname{rot}\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad (\operatorname{rot}\mathbf{E})_{x,z} = 0$$

$$E_y = 0 \qquad E_z = 0$$

$$(\operatorname{rot}\mathbf{E})_y = \frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}$$



$$B_y = (A_0/v)\sin[\omega(t - z/v)]$$

$$E_x = \nu B_y$$

$$E_x = \nu \mu\mu_0 H_y$$

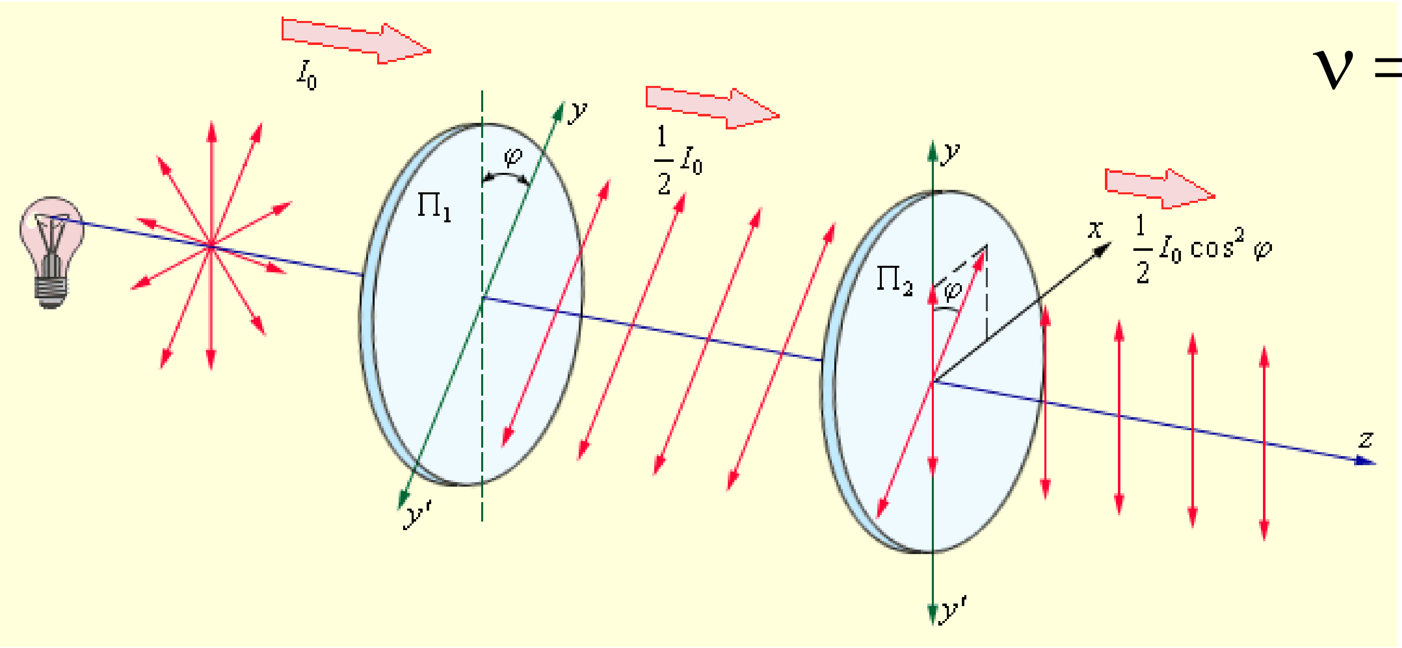
$$\nu = (\mu\mu_0\varepsilon\varepsilon_0)^{-1/2}$$

$$S = <|[\mathbf{EH}]|> = < E_x H_y > = (\nu\mu\mu_0)^{-1} < E_x^2 > =$$

$$= (\varepsilon\varepsilon_0 / \mu\mu_0)^{1/2} A_0^2 < \sin^2[\omega(t - z/v)] > = (\varepsilon\varepsilon_0 / \mu\mu_0)^{1/2} A_0^2 / 2$$

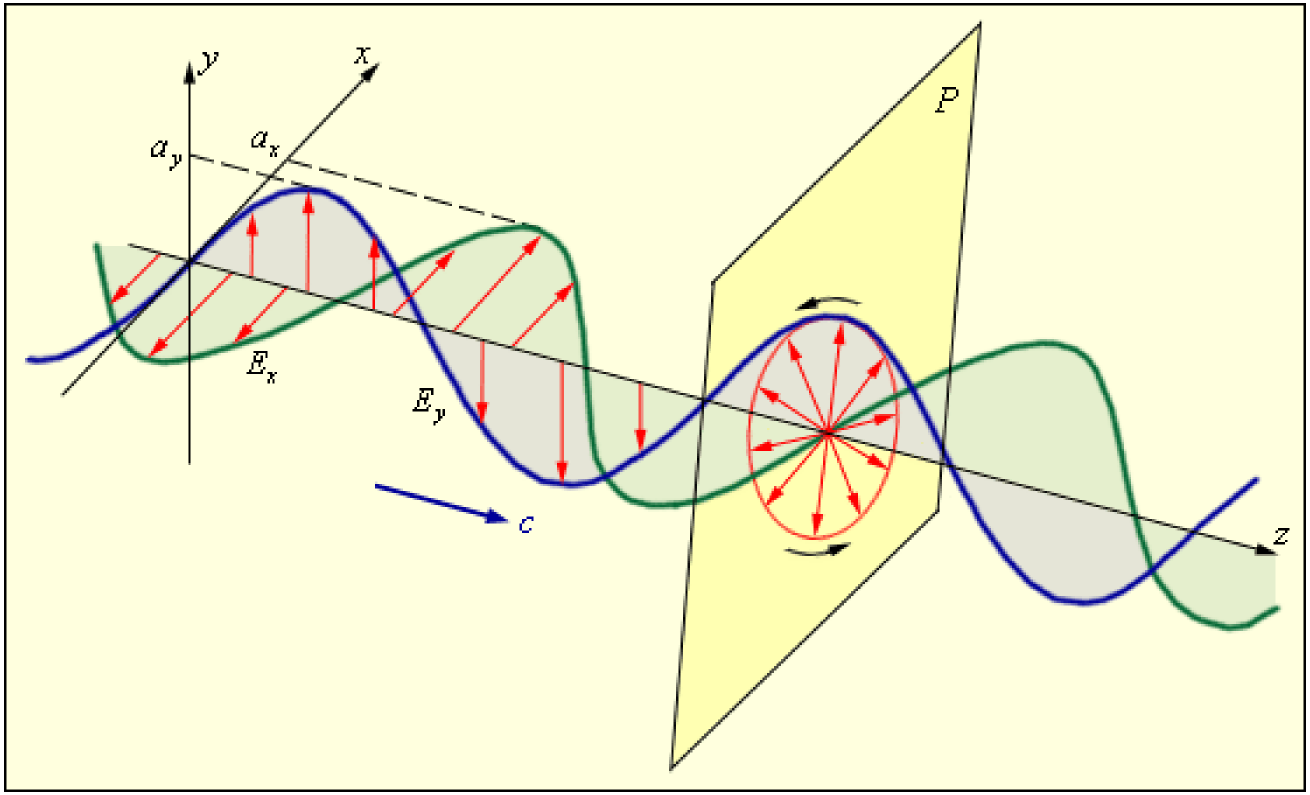
$$w = \frac{1}{2} <|(\mathbf{ED} + \mathbf{BH})|> = \frac{\varepsilon\varepsilon_0}{2} < E^2 > + \frac{1}{2\mu\mu_0} < B^2 > =$$

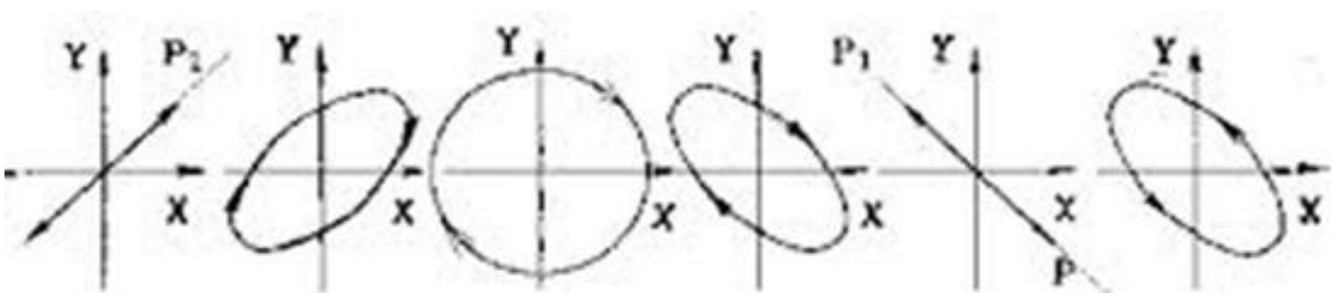
$$= \frac{\varepsilon\varepsilon_0}{2} < E^2 > + \frac{1}{2\mu\mu_0\nu^2} < E^2 > = \varepsilon\varepsilon_0 < E^2 > = \frac{\varepsilon\varepsilon_0}{2} A_0^2$$



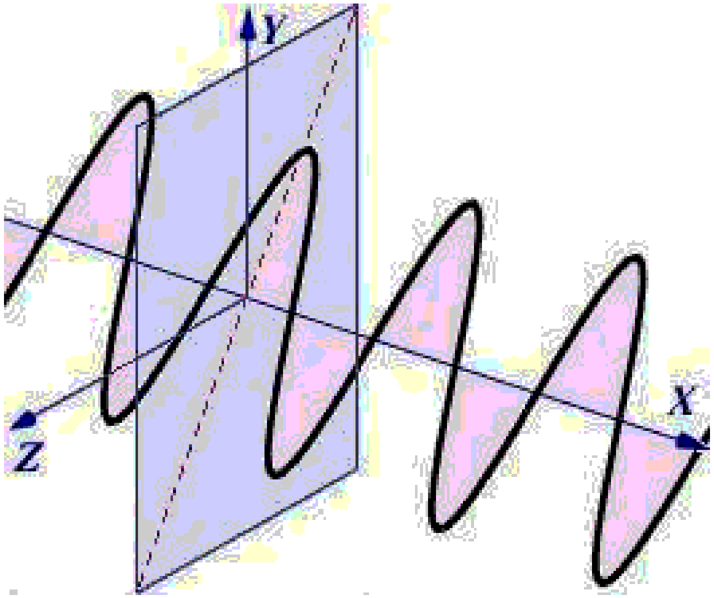
$$\nu = (\mu\mu_0\varepsilon\varepsilon_0)^{-1/2}$$

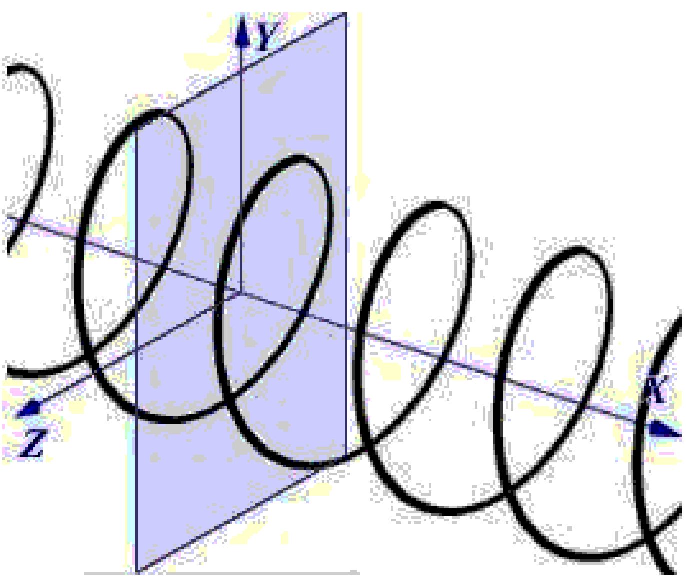
$$S\nu = w$$

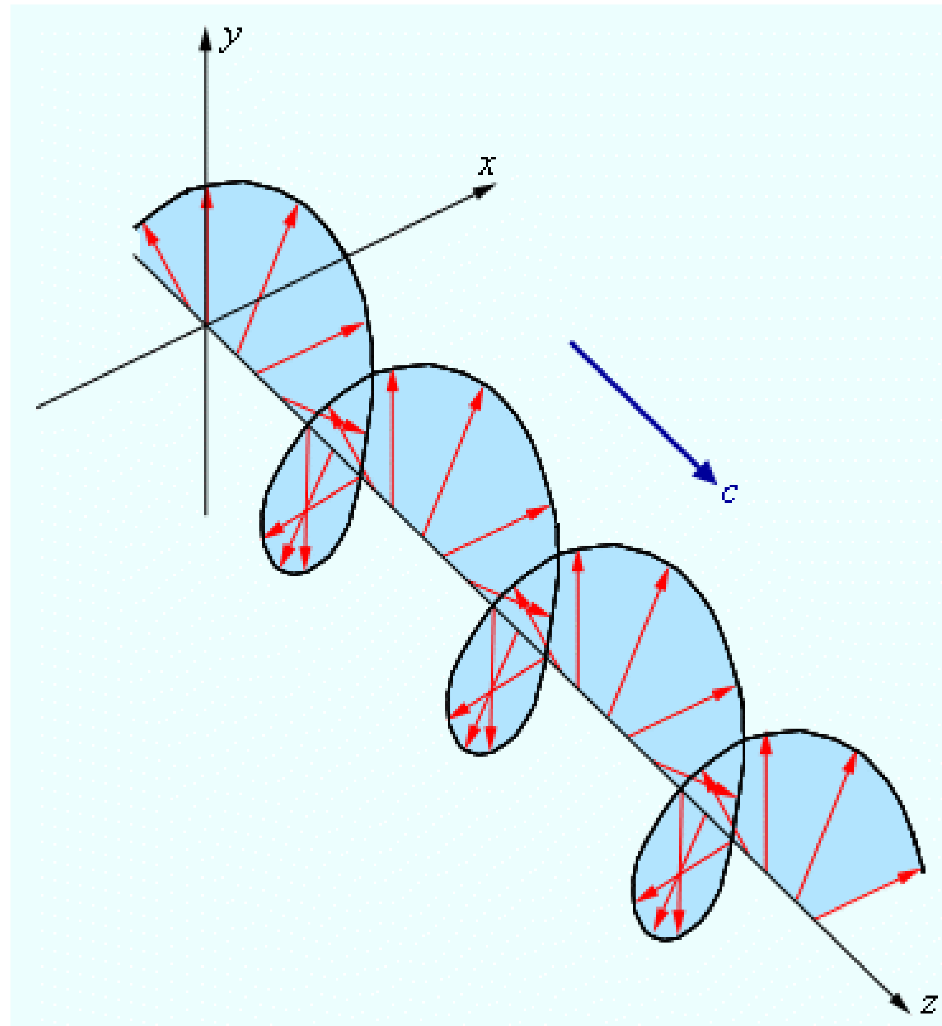


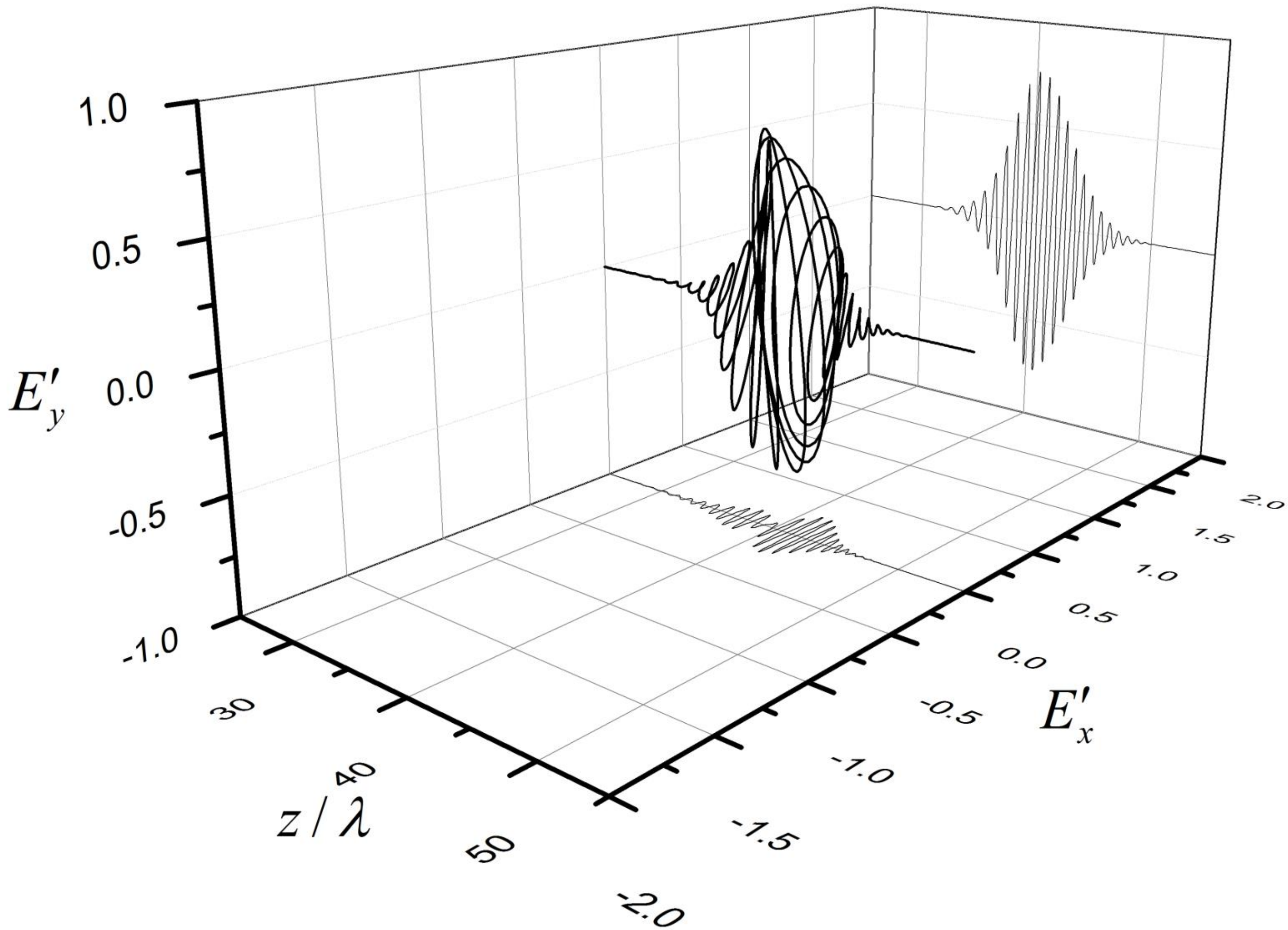


$\Delta\varphi = 0$ $\Delta\varphi = \pi/6$ $\Delta\varphi = \pi/2$ $\Delta\varphi = 5\pi/6$ $\Delta\varphi = \pi$ $\Delta\varphi = 7\pi/6$


Y
X
Z


Y
X
Z


y
x
c
z



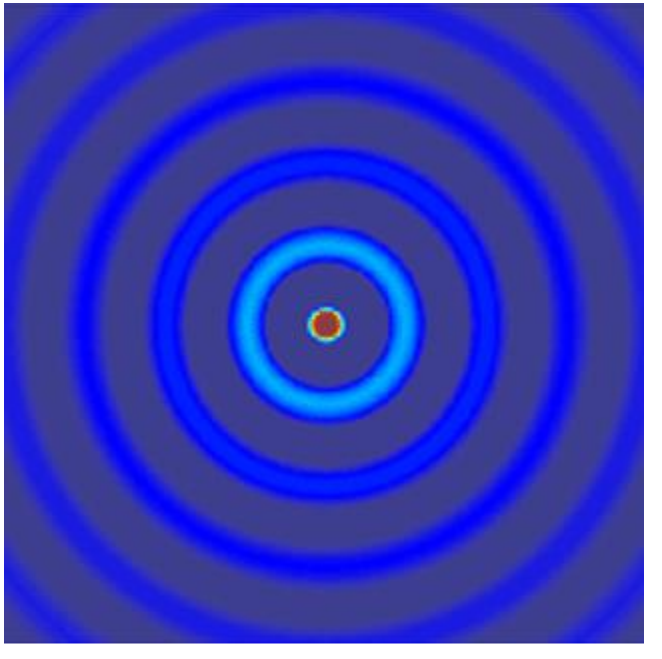
1.0
0.5
0.0
-0.5
-1.0
E'_y
30
40
50
z / λ
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
E'_x

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \mathbf{E}}{\partial r}\right)-\frac{1}{v^2}\frac{\partial^2 \mathbf{E}}{\partial t^2}=0$$

$$\frac{\partial^2 \mathbf{E}}{\partial r^2}+\frac{2}{r}\frac{\partial \mathbf{E}}{\partial r}-\frac{1}{v^2}\frac{\partial^2 \mathbf{E}}{\partial t^2}=0$$

$$\frac{1}{r}\frac{\partial^2 (r\mathbf{E})}{\partial r^2}-\frac{1}{v^2}\frac{\partial^2 \mathbf{E}}{\partial t^2}=0$$

$$\mathbf{E}=\frac{\mathbf{F}_1(t-r/v)}{r}+\frac{\mathbf{F}_2(t+r/v)}{r}$$



$$\mathbf{E}=\frac{\mathbf{A}_1\sin[\omega(t-r/v)+\varphi_1]}{r}$$

Скалярный и векторный потенциалы переменных во времени полей

$$\operatorname{div}\mathbf{B}(\mathbf{r},t)=0 \Rightarrow \mathbf{B}(\mathbf{r},t)=\operatorname{rot}\mathbf{A}(\mathbf{r},t)$$

$$\operatorname{rot}\mathbf{E}(\mathbf{r},t)=-\frac{\partial}{\partial t}\mathbf{B}(\mathbf{r},t)=-\frac{\partial}{\partial t}\operatorname{rot}\mathbf{A}(\mathbf{r},t)=-\operatorname{rot}\frac{\partial\mathbf{A}}{\partial t}$$

$$\operatorname{rot}(\mathbf{E}+\partial\mathbf{A}/\partial t)=0 \Rightarrow \mathbf{E}+\partial\mathbf{A}/\partial t=-\operatorname{grad}\varphi(\mathbf{r},t)$$

$$\boxed{\mathbf{E}=-\partial\mathbf{A}(\mathbf{r},t)/\partial t-\operatorname{grad}\varphi(\mathbf{r},t) \qquad \mathbf{B}=\operatorname{rot}\mathbf{A}(\mathbf{r},t)}$$

$$\mathbf{A}_1=\mathbf{A}+\operatorname{grad}\vartheta(\mathbf{r},t) \qquad \varphi_1=\varphi-\partial\vartheta(\mathbf{r},t)/\partial t$$

$$\operatorname{rot}\mathbf{A}_1(\mathbf{r},t)=\mathbf{B}(\mathbf{r},t) \qquad \mathbf{E}=-\partial\mathbf{A}_1(\mathbf{r},t)/\partial t-\operatorname{grad}\varphi_1(\mathbf{r},t)$$

$$\mathbf{E} = -\partial \mathbf{A}_1(\mathbf{r},t)/\partial t - \operatorname{grad}\varphi_1(\mathbf{r},t) = -\partial \mathbf{A}(\mathbf{r},t)/\partial t -$$

$$-\cancel{\frac{\partial}{\partial t}\operatorname{grad}\vartheta} - \operatorname{grad}\varphi(\mathbf{r},t) + \cancel{\operatorname{grad}\frac{\partial\vartheta}{\partial t}} = -\partial \mathbf{A}(\mathbf{r},t)/\partial t - \operatorname{grad}\varphi(\mathbf{r},t)$$

Условие калибровки Лоренца

$$\boxed{\operatorname{div}\mathbf{A}(\mathbf{r},t) + \mu\mu_0\varepsilon\varepsilon_0\partial\varphi(\mathbf{r},t)/\partial t = 0}$$

Уравнение для скалярного и векторного потенциалов

$$\operatorname{rot}\operatorname{rot}\mathbf{A} = \operatorname{rot}\mathbf{B} = \mu\mu_0\operatorname{rot}\mathbf{H} = \mu\mu_0(\mathbf{j} + \varepsilon\varepsilon_0\partial\mathbf{E}/\partial t) = \mu\mu_0\mathbf{j} +$$

$$+\mu\mu_0\varepsilon\varepsilon_0\frac{\partial}{\partial t}(-\operatorname{grad}\varphi - \partial\mathbf{A}/\partial t) = \mu\mu_0\mathbf{j} - \mu\mu_0\varepsilon\varepsilon_0\frac{\partial^2\mathbf{A}}{\partial t^2} -$$

$$-\mu\mu_0\varepsilon\varepsilon_0\operatorname{grad}(\partial\varphi/\partial t)$$

$$\operatorname{rot}\operatorname{rot}\mathbf{A} = \operatorname{grad}\operatorname{div}\mathbf{A} - \Delta\mathbf{A}$$

$$\mu\mu_0\mathbf{j} - \mu\mu_0\varepsilon\varepsilon_0\frac{\partial^2\mathbf{A}}{\partial t^2}\overbrace{-\mu\mu_0\varepsilon\varepsilon_0\operatorname{grad}(\partial\varphi/\partial t) = \operatorname{grad}\operatorname{div}\mathbf{A}}^{=0} - \Delta\mathbf{A}$$

$$\boxed{\Delta\mathbf{A}-\mu\mu_0\varepsilon\varepsilon_0\frac{\partial^2\mathbf{A}}{\partial t^2}=-\mu\mu_0\mathbf{j}(\mathbf{r},t)}$$

$$\frac{\rho(\mathbf{r},t)}{\varepsilon\varepsilon_0}=\operatorname{div}\mathbf{E}=\operatorname{div}(-\operatorname{grad}\varphi-\partial\mathbf{A}/\partial t)=-\Delta\varphi-\operatorname{div}\frac{\partial}{\partial t}\mathbf{A}=$$

$$=-\Delta\varphi-\frac{\partial}{\partial t}\left(\operatorname{div}\mathbf{A}\right)=-\Delta\varphi-\frac{\partial}{\partial t}\left(-\varepsilon\varepsilon_0\mu\mu_0\frac{\partial\varphi}{\partial t}\right)=$$

$$=-\Delta\varphi+\varepsilon\varepsilon_0\mu\mu_0\frac{\partial^2\varphi}{\partial t^2}$$

Условие калибровки

$$\boxed{\Delta\varphi-\mu\mu_0\varepsilon\varepsilon_0\frac{\partial^2\varphi}{\partial t^2}=-\rho(\mathbf{r},t)/\varepsilon\varepsilon_0}$$

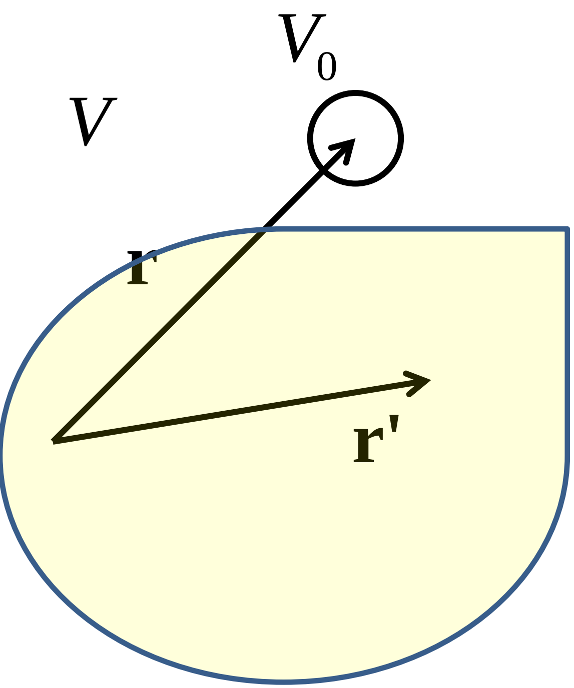
Решение уравнений для скалярного и векторного потенциалов

$$\mathbf{A}(\mathbf{r},t)=\frac{\mu\mu_0}{4\pi}\oint\frac{\mathbf{j}(\mathbf{r}',t-|\mathbf{r}-\mathbf{r}'|/v)}{|\mathbf{r}-\mathbf{r}'|}d\mathbf{r}'$$

$$\varphi(\mathbf{r},t)=\frac{1}{4\pi\varepsilon\varepsilon_0}\oint\frac{\rho(\mathbf{r}',t-|\mathbf{r}-\mathbf{r}'|/v)}{|\mathbf{r}-\mathbf{r}'|}d\mathbf{r}'$$

Надо убедиться, что выписанные выше формулы для скалярного и векторного потенциалов действительно являются решениями волновых уравнений и что они удовлетворяют условию калибровки

$$\Delta\varphi - \frac{1}{v^2}\frac{\partial^2\varphi}{\partial t^2} = -\rho(\mathbf{r},t)/\varepsilon\varepsilon_0 \Rightarrow \varphi(\mathbf{r},t) = \frac{1}{4\pi\varepsilon\varepsilon_0}\oint \frac{\rho(\mathbf{r}', t-|\mathbf{r}-\mathbf{r}'|/v)}{|\mathbf{r}-\mathbf{r}'|}d\mathbf{r}'$$



$$\varphi(\mathbf{r},t) = \underline{\frac{1}{4\pi\varepsilon\varepsilon_0}\int\limits_{V_0\to 0} \frac{\rho(\mathbf{r}', t-|\mathbf{r}-\mathbf{r}'|/v)}{|\mathbf{r}-\mathbf{r}'|}d\mathbf{r}'} +$$

$$+\underline{\frac{1}{4\pi\varepsilon\varepsilon_0}\int\limits_{V} \frac{\rho(\mathbf{r}', t-|\mathbf{r}-\mathbf{r}'|/v)}{|\mathbf{r}-\mathbf{r}'|}d\mathbf{r}'} = \varphi_1 + \varphi_2$$

Оператор Даламбера $\quad \Box = \Delta - \dfrac{1}{v^2}\dfrac{\partial^2}{\partial t^2}$

$\Box\varphi_2 = 0$ Действительно $\mathbf{r}-\mathbf{r}'$ ограничено снизу

$$\Rightarrow \underline{\frac{1}{4\pi\varepsilon\varepsilon_0}\Box\int\limits_{V} \frac{\rho(\mathbf{r}', t-|\mathbf{r}-\mathbf{r}'|/v)}{|\mathbf{r}-\mathbf{r}'|}d\mathbf{r}'} = \frac{1}{4\pi\varepsilon\varepsilon_0}\int\limits_{V} \underbrace{\Box\left(\frac{\rho(\mathbf{r}', t-|\mathbf{r}-\mathbf{r}'|/v)}{|\mathbf{r}-\mathbf{r}'|}\right)}_{=0}d\mathbf{r}' = 0$$

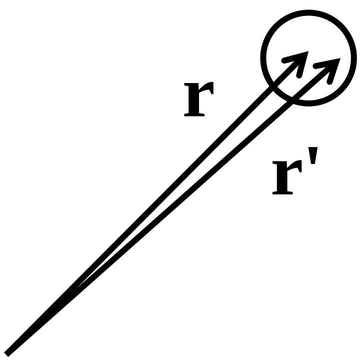
$$\frac{1}{v^2}\frac{\partial^2\varphi_1}{\partial t^2} = \frac{1}{4\pi\varepsilon\varepsilon_0}\frac{1}{v^2}\frac{\partial^2}{\partial t^2}\int\limits_{V_0\to 0}\frac{\rho(\mathbf{r}',t-|\mathbf{r}-\mathbf{r}'|/v)}{|\mathbf{r}-\mathbf{r}'|}d\mathbf{r}' =$$

$$= \frac{1}{4\pi\varepsilon\varepsilon_0}\frac{1}{v^2}\int\limits_{V_0\to 0}\frac{\partial^2}{\partial t^2}\frac{\rho(\mathbf{r}',t-|\mathbf{r}-\mathbf{r}'|/v)}{|\mathbf{r}-\mathbf{r}'|}d\mathbf{r}' = \Rightarrow$$

$$\Rightarrow \quad = \frac{1}{4\pi\varepsilon\varepsilon_0 v^2}\frac{\partial^2}{\partial t^2}\rho(\tilde{\mathbf{r}},t-\tilde{R}/v)\int\limits_{V_0\to 0}\frac{4\pi R^2}{R}dR$$

$$=0$$

$$\tilde{\mathbf{R}} = \tilde{\mathbf{r}} - \mathbf{r}' \qquad \mathbf{R} = \mathbf{r} - \mathbf{r}'$$



$$\Delta\psi = \operatorname{div}\operatorname{grad}\varphi_1 = \lim_{V_0\to 0}\frac{1}{V_0}\oint\limits_{S_0}\operatorname{grad}(\varphi_1)d\mathbf{S}_0 \Rightarrow$$

$$\operatorname{grad}\varphi_1 = \operatorname{grad}\left(\frac{1}{4\pi\varepsilon\varepsilon_0}\int\limits_{V_0\to 0}\frac{\rho(\mathbf{r}',t-|\mathbf{r}-\mathbf{r}'|/v)}{|\mathbf{r}-\mathbf{r}'|}d\mathbf{r}'\right)$$

$$\operatorname{grad}\varphi_1 = \operatorname{grad}\left(\frac{1}{4\pi\varepsilon\varepsilon_0}\int\limits_{V_0\to 0}\frac{\rho(\mathbf{r}',t-|\mathbf{r}-\mathbf{r}'|/v)}{|\mathbf{r}-\mathbf{r}'|}d\mathbf{r}'\right) =$$

$$= \frac{1}{4\pi\varepsilon\varepsilon_0}\int\limits_{V_0\to 0}\operatorname{grad}\left(\frac{\rho(\mathbf{r}',t-|\mathbf{r}-\mathbf{r}'|/v)}{|\mathbf{r}-\mathbf{r}'|}\right)d\mathbf{r}' =$$

$$= \underbrace{\frac{1}{4\pi\varepsilon\varepsilon_0}\int\limits_{V_0\to 0}\frac{\operatorname{grad}\{\rho(\mathbf{r}',t-|\mathbf{r}-\mathbf{r}'|/v)\}}{|\mathbf{r}-\mathbf{r}'|}d\mathbf{r}'}_{=0} - \underline{\frac{1}{4\pi\varepsilon\varepsilon_0}\int\limits_{V_0\to 0}\rho(\mathbf{r}',t-|\mathbf{r}-\mathbf{r}'|/v)\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}d\mathbf{r}'}$$

т.к. $\int\limits_{V_0\to 0}\frac{\operatorname{grad}\{\rho(\mathbf{r}',t-|\mathbf{r}-\mathbf{r}'|/v)\}}{|\mathbf{r}-\mathbf{r}'|}d\mathbf{r}' = \operatorname{grad}\rho(\tilde{\mathbf{r}},t-\tilde{R}/v)\int\limits_{V_0\to 0}\frac{4\pi R^2 dR}{R|}\to 0$

$\mathbf{R} = \mathbf{r}-\mathbf{r}'$

$\tilde{\mathbf{R}} = \tilde{\mathbf{r}}-\mathbf{r}'$

$$\operatorname{div}\operatorname{grad}\varphi_1 = \lim_{V_0\to 0}\frac{1}{V_0}\oint\limits_{S_0}\underline{\operatorname{grad}(\varphi_1)}d\mathbf{S_0} = -\frac{1}{4\pi\varepsilon\varepsilon_0}\times$$

$$\times\lim_{V_0\to 0}\frac{1}{V_0}\oint\limits_{S_0}\int\limits_{V_0\to 0}\frac{\rho(\mathbf{r}',t-|\mathbf{r}-\mathbf{r}'|/v)(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}d\mathbf{S_0}d\mathbf{r}' =$$

$$= -\frac{1}{4\pi\varepsilon\varepsilon_0}\times\lim_{V_0\to 0}\frac{1}{V_0}\int\limits_{V_0\to 0}\rho(\mathbf{r}',t-|\mathbf{r}-\mathbf{r}'|/v)d\mathbf{r}'\int\limits_{S_0}\frac{(\mathbf{r}-\mathbf{r}')d\mathbf{S_0}}{|\mathbf{r}-\mathbf{r}'|^3}$$

$= 4\pi$

Меняем порядок интегрирования

$$\Delta\varphi_1 = -\frac{1}{\varepsilon\varepsilon_0}\times\lim_{V_0\to 0}\frac{1}{V_0}\int\limits_{V_0\to 0}\rho(\mathbf{r}',t-|\mathbf{r}-\mathbf{r}'|/v)d\mathbf{r}' =$$

$$= -\frac{1}{\varepsilon\varepsilon_0}\times\lim_{V_0\to 0}\rho(\tilde{\mathbf{r}},t-\tilde{R}/v)\frac{1}{V_0}\int\limits_{V_0\to 0}d\mathbf{r}' =$$

$$\mathbf{R} = \mathbf{r}-\mathbf{r}'$$

$$\tilde{\mathbf{R}} = \tilde{\mathbf{r}}-\mathbf{r}'$$

$$= -\frac{1}{\varepsilon\varepsilon_0}\times\lim_{V_0\to 0}\rho(\tilde{\mathbf{r}},t-\tilde{R}/v) =$$

$$= -\frac{1}{\varepsilon\varepsilon_0}\times\lim_{V_0\to 0}\rho(\mathbf{r},t-|\tilde{\mathbf{r}}-\mathbf{r}'|/v) = -\frac{1}{\varepsilon\varepsilon_0}\rho(\mathbf{r},t)$$

$$V_0\to 0 \quad\leftrightarrow\quad \tilde{\mathbf{R}} = \tilde{\mathbf{r}}-\mathbf{r}' \quad\to 0$$

$$\Delta\varphi-\mu\mu_0\varepsilon\varepsilon_0\frac{\partial^2\varphi}{\partial t^2} = -\rho(\mathbf{r},t)/\varepsilon\varepsilon_0 \qquad \varphi(\mathbf{r},t) = \frac{1}{4\pi\varepsilon\varepsilon_0}\oint\frac{\rho(\mathbf{r}',t-|\mathbf{r}-\mathbf{r}'|/v)}{|\mathbf{r}-\mathbf{r}'|}d\mathbf{r}'$$

$$\Delta\mathbf{A}-\mu\mu_0\varepsilon\varepsilon_0\frac{\partial^2\mathbf{A}}{\partial t^2} = -\mu\mu_0\mathbf{j}(\mathbf{r},t) \qquad \mathbf{A}(\mathbf{r},t) = \frac{\mu\mu_0}{4\pi}\oint\frac{\mathbf{j}(\mathbf{r}',t-|\mathbf{r}-\mathbf{r}'|/v)}{|\mathbf{r}-\mathbf{r}'|}d\mathbf{r}'$$

$$\operatorname{div}\mathbf{A}(\mathbf{r},t)+\mu\mu_0\varepsilon\varepsilon_0\partial\varphi(\mathbf{r},t)/\partial t=0$$

$$\operatorname{div}\mathbf{A}(\mathbf{r},t)=\frac{\mu\mu_0}{4\pi}\oint\operatorname{div}\left(\frac{\mathbf{j}(\mathbf{r}',t-|\mathbf{r}-\mathbf{r}'|/v)}{|\mathbf{r}-\mathbf{r}'|}\right)d\mathbf{r}'$$

$$\operatorname{div}\psi\mathbf{a}=\psi\operatorname{div}\mathbf{a}+\mathbf{a}\cdot\operatorname{grad}\psi \qquad \operatorname{div}\mathbf{F}(f(\mathbf{r}))=\frac{\partial\mathbf{F}}{\partial f}\operatorname{grad}f(\mathbf{r})$$

$$t'(\mathbf{r})=t-|\mathbf{r}-\mathbf{r}'|/v$$

$$\operatorname{div}\left(\frac{\mathbf{j}(\mathbf{r}',t')}{|\mathbf{r}-\mathbf{r}'|}\right)=\frac{1}{|\mathbf{r}-\mathbf{r}'|}\operatorname{div}\mathbf{j}(\mathbf{r}',t')+\mathbf{j}(\mathbf{r}',t')\operatorname{grad}\frac{1}{|\mathbf{r}-\mathbf{r}'|}=$$

$$=\underline{\frac{1}{|\mathbf{r}-\mathbf{r}'|}\frac{\partial\mathbf{j}(\mathbf{r}',t')}{\partial t'}\left(-\frac{1}{v}\right)\operatorname{grad}|\mathbf{r}-\mathbf{r}'|}+\underline{\mathbf{j}(\mathbf{r}',t')\operatorname{grad}\frac{1}{|\mathbf{r}-\mathbf{r}'|}}$$

$$\operatorname{div}'\left(\frac{\mathbf{j}(\mathbf{r}',t')}{|\mathbf{r}-\mathbf{r}'|}\right)=\frac{1}{|\mathbf{r}-\mathbf{r}'|}\operatorname{div}'\mathbf{j}(\mathbf{r}',t')_{t'=\mathrm{const}}+$$

$$+\underline{\frac{1}{|\mathbf{r}-\mathbf{r}'|}\frac{\partial\mathbf{j}(\mathbf{r},t')}{\partial t'}\left(-\frac{1}{v}\right)\operatorname{grad}'|\mathbf{r}-\mathbf{r}'|}+\underline{\mathbf{j}(\mathbf{r},t')\cdot\operatorname{grad}'\frac{1}{|\mathbf{r}-\mathbf{r}'|}}$$

Сложем две дивергенции. Подчеркнутые одним цветом слагаемые отличаются знаком

Сложим две дивергенции

$$\operatorname{div}\left(\frac{\mathbf{j}(\mathbf{r}',t')}{|\mathbf{r}-\mathbf{r}'|}\right)+\operatorname{div}'\left(\frac{\mathbf{j}(\mathbf{r}',t')}{|\mathbf{r}-\mathbf{r}'|}\right)=\frac{1}{|\mathbf{r}-\mathbf{r}'|}\operatorname{div}'\mathbf{j}(\mathbf{r}',t')_{t'=\mathrm{const}}$$

$$\operatorname{div}\mathbf{A}(\mathbf{r},t)=\frac{\mu\mu_0}{4\pi}\oint\operatorname{div}\left(\frac{\mathbf{j}(\mathbf{r}',t')}{|\mathbf{r}-\mathbf{r}'|}\right)d\mathbf{r}'=-\frac{\mu\mu_0}{4\pi}\oint\operatorname{div}'\left(\frac{\mathbf{j}(\mathbf{r}',t')}{|\mathbf{r}-\mathbf{r}'|}\right)d\mathbf{r}'+$$

$$+\frac{\mu\mu_0}{4\pi}\oint\frac{1}{|\mathbf{r}-\mathbf{r}'|}\operatorname{div}'\mathbf{j}(\mathbf{r}',t')_{t'=\mathrm{const}}d\mathbf{r}'=$$

$$t'=t-|\mathbf{r}-\mathbf{r}'|/v$$

$$=\underbrace{-\frac{\mu\mu_0}{4\pi}\oint_S\frac{j_n(\mathbf{r}',t')}{|\mathbf{r}-\mathbf{r}'|}dS}_{=0}+\frac{\mu\mu_0}{4\pi}\oint\frac{1}{|\mathbf{r}-\mathbf{r}'|}\operatorname{div}'\mathbf{j}(\mathbf{r}',t')_{t'=\mathrm{const}}d\mathbf{r}'$$

Вычислим теперь

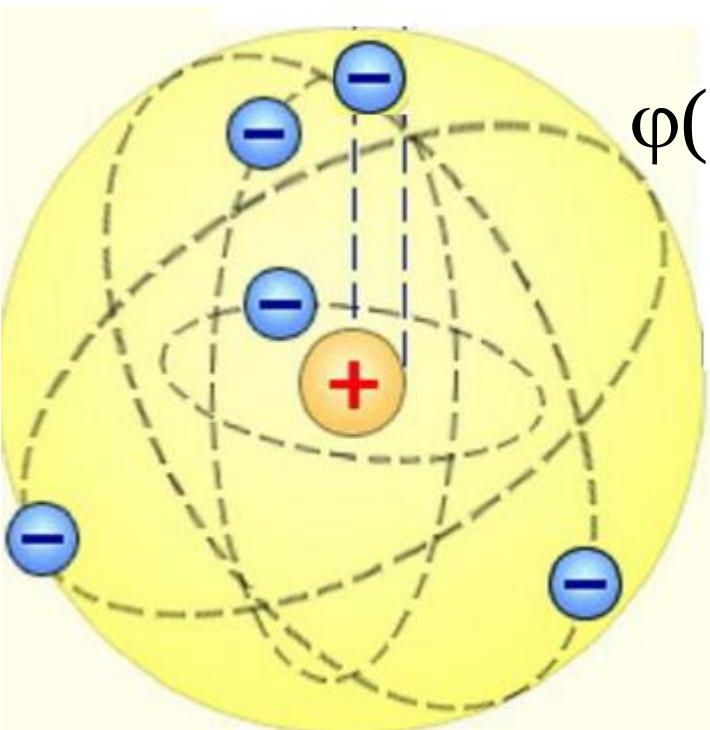
$$\mu\mu_0\varepsilon\varepsilon_0\frac{\partial\varphi(\mathbf{r},t)}{\partial t}=\frac{\mu\mu_0}{4\pi}\frac{\partial}{\partial t}\oint\frac{\rho(\mathbf{r}',t')}{|\mathbf{r}-\mathbf{r}'|}d\mathbf{r}'\overset{\partial/\partial t'=\partial/\partial t}{=}\frac{\mu\mu_0}{4\pi}\oint\frac{1}{|\mathbf{r}-\mathbf{r}'|}\frac{\partial\rho(\mathbf{r}',t')}{\partial t'}d\mathbf{r}'$$

Сложим подчеркнутые красной чертой слагаемые

$$\operatorname{div}\mathbf{A}(\mathbf{r},t)+\mu\mu_0\varepsilon\varepsilon_0\frac{\partial\varphi(\mathbf{r},t)}{\partial t}=$$

$$=\frac{\mu\mu_0}{4\pi}\oint\left\{\frac{\partial\rho(\mathbf{r}',t')}{\partial t'}+\operatorname{div}'\mathbf{j}(\mathbf{r}',t')\right\}\frac{d\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}=0$$

$= 0$



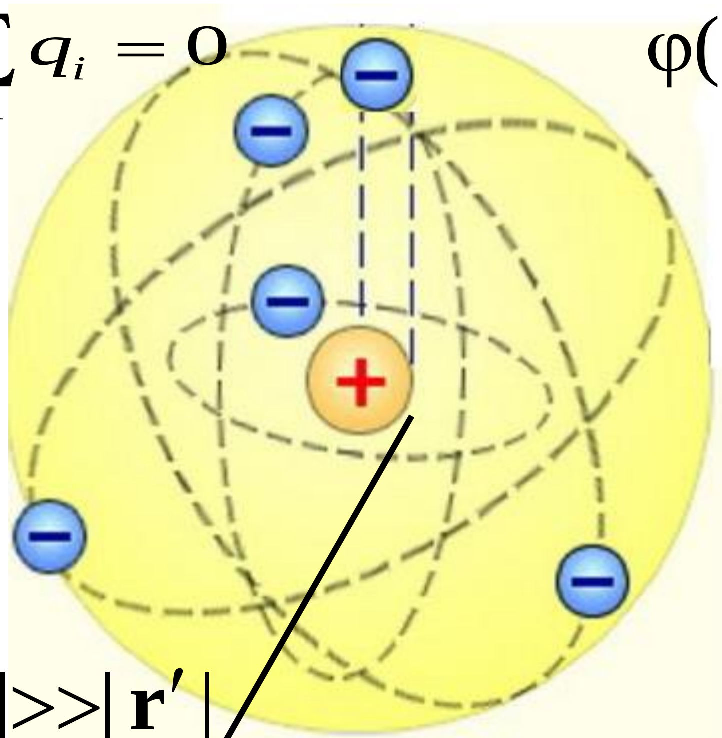
$$\varphi(\mathbf{r},t)=\frac{1}{4\pi\varepsilon\varepsilon_0}\oint\frac{\rho(\mathbf{r}',t-|\mathbf{r}-\mathbf{r}'|/v)}{|\mathbf{r}-\mathbf{r}'|}d\mathbf{r}'$$

$$\mathbf{A}(\mathbf{r},t)=\frac{\mu\mu_0}{4\pi}\oint\frac{\mathbf{j}(\mathbf{r}',t-|\mathbf{r}-\mathbf{r}'|/v)}{|\mathbf{r}-\mathbf{r}'|}d\mathbf{r}'$$

$$\mathbf{E}=-\partial\mathbf{A}(\mathbf{r},t)/\partial t-\operatorname{grad}\varphi(\mathbf{r},t)$$

$$\mathbf{B}(\mathbf{r},t)=\operatorname{rot}\mathbf{A}(\mathbf{r},t)$$

$$\sum_{i=1}^{n} q_i = 0$$



$$|\mathbf{r}| >> |\mathbf{r}'|$$

$\mathbf{r}$

$$\varphi(\mathbf{r},t) = \frac{1}{4\pi\varepsilon\varepsilon_0}\oint \frac{\rho(\mathbf{r}', t - |\mathbf{r}-\mathbf{r}'|/v)}{|\mathbf{r}-\mathbf{r}'|}d\mathbf{r}'$$

$$|\mathbf{r}-\mathbf{r}'| \approx r - \frac{\mathbf{r}\mathbf{r}'}{r}$$

$$\varphi(\mathbf{r},t) \approx \frac{1}{4\pi\varepsilon\varepsilon_0 r}\underbrace{\oint \rho(\mathbf{r}', t - r/v)d\mathbf{r}'}_{=0} -$$

$$-\frac{1}{4\pi\varepsilon\varepsilon_0}\oint \frac{\mathbf{r}\cdot\mathbf{r}'}{r}\frac{\partial}{\partial r}\left(\frac{\rho(\mathbf{r}', t - r/v)}{r}\right)d\mathbf{r}'$$

$$\varphi(\mathbf{r},t) \approx -\frac{1}{4\pi\varepsilon\varepsilon_0}\frac{\mathbf{r}}{r}\frac{\partial}{\partial r}\left(\frac{1}{r}\oint \mathbf{r}'\rho(\mathbf{r}', t - r/v)d\mathbf{r}'\right) =$$

$$= -\frac{1}{4\pi\varepsilon\varepsilon_0}\frac{\mathbf{r}}{r}\frac{\partial}{\partial r}\left(\frac{\mathbf{p}(t - r/v)}{r}\right) = -\frac{1}{4\pi\varepsilon\varepsilon_0}\left(-\frac{\mathbf{p}\cdot\mathbf{r}}{r^3} + \frac{\mathbf{r}}{r^2}\frac{\partial\mathbf{p}}{\partial r}\right) =$$

$$= -\frac{1}{4\pi\varepsilon\varepsilon_0}\left(\mathbf{p}\cdot\mathrm{grad}\left(\frac{1}{r}\right) + \frac{1}{r}\mathrm{div}\,\mathbf{p}\right) \Rightarrow$$

?

$$\frac{\mathbf{r}}{r}\frac{\partial \mathbf{p}}{\partial r}=\frac{x}{r}\frac{\partial p_x}{\partial r}+\frac{y}{r}\frac{\partial p_y}{\partial r}+\frac{z}{r}\frac{\partial p_z}{\partial r}=\frac{\partial r}{\partial x}\frac{\partial p_x}{\partial r}+\frac{\partial r}{\partial y}\frac{\partial p_y}{\partial r}+\frac{\partial r}{\partial z}\frac{\partial p_z}{\partial r}=\operatorname{div}\mathbf{p}$$

$$\varphi(\mathbf{r},t)\approx-\frac{1}{4\pi\varepsilon\varepsilon_0}\left(\mathbf{p}\cdot\operatorname{grad}\left(\frac{1}{r}\right)+\frac{1}{r}\operatorname{div}\mathbf{p}\right)=-\frac{1}{4\pi\varepsilon\varepsilon_0}\operatorname{div}\left(\frac{\mathbf{p}(t-r/v)}{r}\right)$$

$$\operatorname{div}\alpha\mathbf{a}=\mathbf{a}\cdot\operatorname{grad}\alpha+\alpha\operatorname{div}\mathbf{a}$$

$$\mathbf{A}(\mathbf{r},t)=\frac{\mu\mu_0}{4\pi}\oint\frac{\mathbf{j}(\mathbf{r}',t-|\mathbf{r}-\mathbf{r}'|/v)}{|\mathbf{r}-\mathbf{r}'|}d\mathbf{r}'\approx\frac{\mu\mu_0}{4\pi}\oint\frac{\mathbf{j}(\mathbf{r}',t-r/v)}{r}d\mathbf{r}'=$$

$$=\frac{\mu\mu_0}{4\pi r}\oint\mathbf{j}(\mathbf{r}',t-r/v)d\mathbf{r}'\Rightarrow \textit{приводится к}\Rightarrow\frac{\mu\mu_0}{4\pi r}\frac{\partial\mathbf{p}(t-r/v)}{\partial t}$$

Уравнение непрерывности $\Rightarrow$ $$\frac{\partial\rho(\mathbf{r}',t-r/v)}{\partial t}=-\operatorname{div}'\mathbf{j}(\mathbf{r}',t-r/v)$$

Умножим на **r** и проинтегрируем

$$\int_V\mathbf{r}'\frac{\partial\rho(\mathbf{r}',t-r/v)}{\partial t}d\mathbf{r}'=\underline{\frac{\partial}{\partial t}\mathbf{p}(t-r/v)}=\underline{-\int_V\mathbf{r}'\operatorname{div}'\mathbf{j}(\mathbf{r}',t-r/v)d\mathbf{r}'}$$

Умножим подчеркнутые члены на постоянный вектор **a** и проинтегрируем

$$\mathbf{a}\frac{\partial}{\partial t}\mathbf{p} = -\mathbf{a}\int_V \mathbf{r}'\,\mathrm{div}'\,\mathbf{j}(\mathbf{r}',t-r/v)d\mathbf{r}' = -\int_V \mathbf{a}\mathbf{r}'\,\mathrm{div}'\,\mathbf{j}(\mathbf{r}',t-r/v)d\mathbf{r}' =$$

$$= -\int_V \mathrm{div}'\big((\mathbf{a}\cdot\mathbf{r}')\mathbf{j}\big)d\mathbf{r}' + \int_V \mathbf{j}\,\mathrm{grad}'\big(\mathbf{a}\cdot\mathbf{r}'\big)d\mathbf{r}' =$$

$$\mathrm{grad}'(\mathbf{a}\cdot\mathbf{r}') = \mathbf{a}$$

$$= \underbrace{-\int_S \big(\mathbf{a}\cdot\mathbf{r}'\big)\,j_n d\mathbf{r}'}_{=0} + \mathbf{a}\int_V \mathbf{j}(\mathbf{r}',t-r/v)d\mathbf{r}'$$

$$\mathrm{div}\,\alpha\mathbf{a} = \mathbf{a}\cdot\mathrm{grad}\,\alpha + \alpha\,\mathrm{div}\,\mathbf{a}$$

$$\boxed{\varphi(\mathbf{r},t) \approx -\frac{1}{4\pi\varepsilon\varepsilon_0}\mathrm{div}\left(\frac{\mathbf{p}(t-r/v)}{r}\right) \qquad \mathbf{A}(\mathbf{r},t) \approx \frac{\mu\mu_0}{4\pi r}\frac{\partial\mathbf{p}(t-r/v)}{\partial t}}$$

Вычислим **B** и **E**

$$\frac{\mathbf{p}(t-r/v)}{r} = \mathbf{p}_0 f(\mathbf{r},t)$$

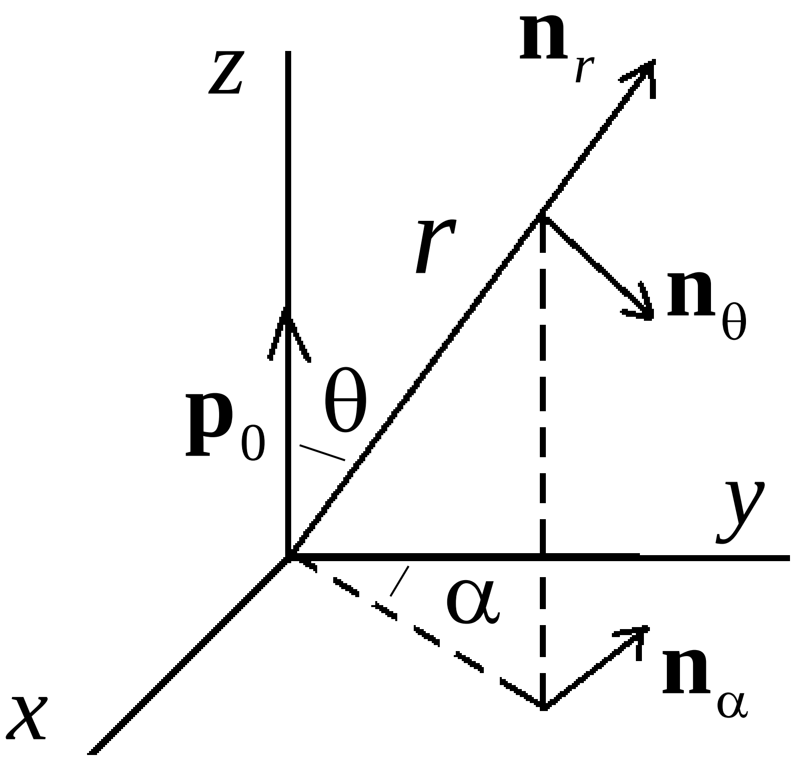
$$\varphi(\mathbf{r},t) \approx -\frac{1}{4\pi\varepsilon\varepsilon_0}\mathrm{div}\,\mathbf{p}_0 f(\mathbf{r},t) \qquad \mathbf{A}(\mathbf{r},t) \approx \frac{\mu\mu_0}{4\pi}\frac{\partial}{\partial t}\mathbf{p}_0 f(\mathbf{r},t)$$

$$\mathbf{B} = \mathrm{rot}\,\mathbf{A} = \frac{\mu\mu_0}{4\pi}\mathrm{rot}\frac{\partial}{\partial t}\mathbf{p}_0 f(\mathbf{r},t) = \frac{\mu\mu_0}{4\pi}\frac{\partial}{\partial t}\mathrm{rot}\,\mathbf{p}_0 f(\mathbf{r},t)$$

$$\mathbf{E} = -\operatorname{grad}\varphi - \partial \mathbf{A}/\partial t = \frac{1}{4\pi\varepsilon\varepsilon_0}\operatorname{grad}\operatorname{div}\mathbf{p}_0 f(r,t) - \frac{\partial}{\partial t}\frac{\mu\mu_0}{4\pi}\frac{\partial}{\partial t}\mathbf{p}_0 f(r,t) =$$

$$= \frac{1}{4\pi\varepsilon\varepsilon_0}\left(\operatorname{rot}\operatorname{rot}\mathbf{p}_0 f(r,t) + \underbrace{\Delta\mathbf{p}_0 f(r,t) - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\mathbf{p}_0 f(r,t)}_{=0}\right) =$$

$$= \frac{1}{4\pi\varepsilon\varepsilon_0}\operatorname{rot}\operatorname{rot}\mathbf{p}_0 f(r,t) \qquad \frac{\mathbf{p}(t - r/v)}{r} = \mathbf{p}_0 f(\mathbf{r},t)$$



$$p_{0r} = p_0\cos\theta$$

$$p_{0\theta} = -p_0\sin\theta$$

$$p_{0\alpha} = 0$$

$$\frac{\mathbf{p}(t - r/v)}{r} = \mathbf{p}_0 f(\mathbf{r},t) = \mathbf{p}_0\frac{\exp[i\omega(t - r/v)]}{r}$$

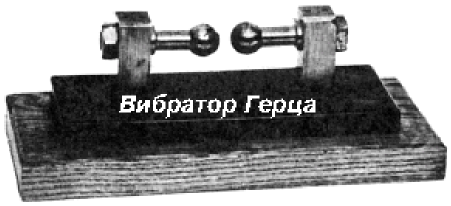
$$\operatorname{rot}\mathbf{p}_0 f = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\cdot p_{0\alpha} f) - \frac{\partial}{\partial\alpha}(p_{0\theta} f)\right]\mathbf{n}_r +$$

$$+\frac{1}{r}\left[\sin\theta\frac{\partial}{\partial\alpha}(p_{0r} f) - \frac{\partial}{\partial r}(rp_{0\alpha} f)\right]\mathbf{n}_\theta +$$

$$+\frac{1}{r}\left[\frac{\partial}{\partial r}(rp_{0\theta} f) - \frac{\partial}{\partial\theta}(p_{0r} f)\right]\mathbf{n}_\alpha$$

$$p_{0r} = p_0\cos\theta$$
$$p_{0\theta} = -p_0\sin\theta$$
$$p_{0\alpha} = 0$$

$$f(\mathbf{r},t) = \frac{\exp[i\omega(t - r/v)]}{r} \qquad \frac{\partial}{\partial r} f(\mathbf{r},t) = -\frac{1}{r} f - \frac{i\omega}{v} f$$



$$\Rightarrow \qquad \left(\operatorname{rot}\mathbf{p}_0 f\right)_r = 0 \qquad \left(\operatorname{rot}\mathbf{p}_0 f\right)_\theta = 0$$

$$\left(\operatorname{rot}\mathbf{p}_0 f\right)_\alpha = \frac{1}{r}\left[-\frac{\partial}{\partial r}(rp_0\sin\theta\cdot f) + p_0\sin\theta\cdot f\right] = -p_0\sin\theta\frac{\partial f}{\partial r} =$$

$$= \frac{p_0\sin\theta}{r}\left(\frac{1}{r} + \frac{i\omega}{v}\right)\exp\left(i\omega(t - r/v)\right)$$

$$\mathrm{rot}\,\mathrm{rot}\,\mathbf{p}_0 f = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\sin\theta\,\underline{(\mathrm{rot}\,\mathbf{p}_0 f)_\alpha}\right) - \frac{\partial}{\partial\alpha}\cancel{(\mathrm{rot}\,\mathbf{p}_0 f)_\theta}\right]\mathbf{n}_r +$$

$$+\frac{1}{r}\left[\sin\theta\,\cancel{\frac{\partial}{\partial\alpha}(\mathrm{rot}\,\mathbf{p}_0 f)_r} - \frac{\partial}{\partial r}\left(r\,\underline{(\mathrm{rot}\,\mathbf{p}_0 f)_\alpha}\right)\right]\mathbf{n}_\theta +$$

$$+\frac{1}{r}\left[\cancel{\frac{\partial}{\partial r}\left(r(\mathrm{rot}\,\mathbf{p}_0 f)_\theta\right)} - \cancel{\frac{\partial}{\partial\theta}(\mathrm{rot}\,\mathbf{p}_0 f)_r}\right]\mathbf{n}_\alpha$$

$$\left(\mathrm{rot}\,\mathbf{p}_0 f\right)_r = 0$$

$$\left(\mathrm{rot}\,\mathbf{p}_0 f\right)_\theta = 0$$

$$\mathbf{E} = \frac{1}{4\pi\varepsilon\varepsilon_0}\mathrm{rotrot}\,\mathbf{p}_0 f(r,t) \quad\Rightarrow\quad E_\alpha = 0$$

$$E_r = \frac{1}{4\pi\varepsilon\varepsilon_0 r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\,\underbrace{\frac{p_0\sin\theta}{r}\left(\frac{1}{r}+\frac{i\omega}{v}\right)\exp\left(i\omega(t-r/v)\right)}_{\left(\mathrm{rot}\,\mathbf{p}_0 f\right)_\alpha = 0}\right) =$$

$$= \frac{p_0\cos\theta}{2\pi\varepsilon\varepsilon_0 r^2}\left(\frac{1}{r}+\frac{i\omega}{v}\right)\exp\left(i\omega(t-r/v)\right)$$

$$E_\theta = -\frac{1}{4\pi\varepsilon\varepsilon_0 r}\frac{\partial}{\partial r}\left(r \cdot \underline{\frac{p_0 \sin\theta}{r}\left(\frac{1}{r}+\frac{i\omega}{v}\right)\exp\left(i\omega(t-r/v)\right)}\right) =$$

$$= -\frac{p_0 \sin\theta}{4\pi\varepsilon\varepsilon_0 r}\left(-\frac{1}{r^2}-\frac{i\omega}{v}\left(\frac{1}{r}+\frac{i\omega}{v}\right)\right)\exp\left(i\omega(t-r/v)\right) =$$

$$= \frac{p_0 \sin\theta}{4\pi\varepsilon\varepsilon_0 r}\left(\frac{1}{r^2}+\frac{i\omega}{rv}-\frac{\omega^2}{v^2}\right)\exp\left(i\omega(t-r/v)\right)$$

$$\boxed{\begin{aligned}\left(\operatorname{rot}\mathbf{p}_0 f\right)_r &= 0\\ \left(\operatorname{rot}\mathbf{p}_0 f\right)_\theta &= 0\end{aligned}}$$

$$\mathbf{B} = \frac{\mu\mu_0}{4\pi}\frac{\partial}{\partial t}\operatorname{rot}\mathbf{p}_0 f(\mathbf{r},t)$$

$$\boxed{\left(\operatorname{rot}\mathbf{p}_0 f\right)_\alpha = \frac{p_0 \sin\theta}{r}\left(\frac{1}{r}+\frac{i\omega}{v}\right)\exp\left(i\omega(t-r/v)\right)}$$

$$B_\alpha = \frac{\mu\mu_0}{4\pi}\frac{\partial}{\partial t}\left(\frac{p_0 \sin\theta}{r}\left(\frac{1}{r}+\frac{i\omega}{v}\right)\exp\left(i\omega(t-r/v)\right)\right) =$$

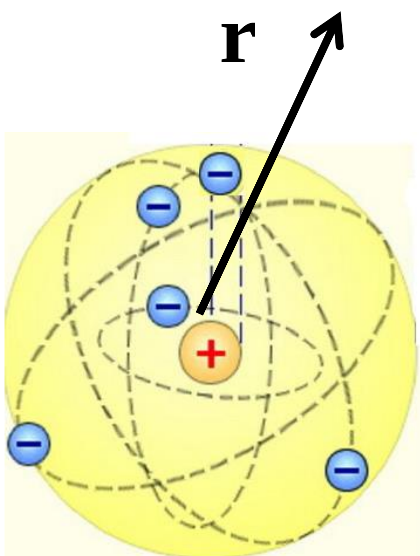
$$B_r = 0$$

$$= \frac{i\mu\mu_0\omega p_0 \sin\theta}{4\pi r v}\left(\frac{1}{r}+\frac{i\omega}{v}\right)\exp\left(i\omega(t-r/v)\right)$$

$$B_\theta = 0$$

$$\mathbf{B} = \{0, 0, B_\alpha\} \qquad \mathbf{E} = \{E_r, E_\theta, 0\} \quad \Longrightarrow \quad \mathbf{B} \perp \mathbf{E}$$

$$\boxed{\lambda << r \quad \Leftrightarrow \quad \frac{1}{r} << \frac{\omega}{v}}$$



$$|\mathbf{r}| >> |\mathbf{r}'|$$

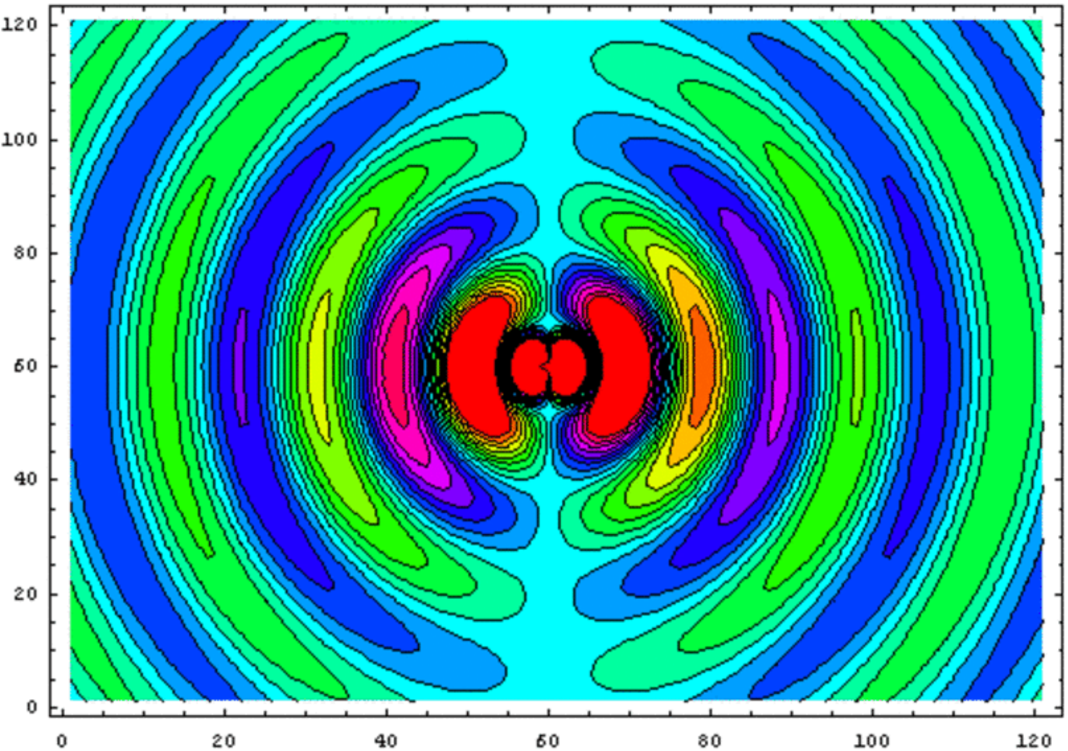
$$H_\alpha = -\frac{\omega^2 p_0 \sin\theta}{4\pi r v} \exp\left(i\omega(t - r/v)\right)$$

$$E_\theta = -\frac{p_0 \omega^2 \sin\theta}{4\pi\varepsilon\varepsilon_0 r v^2} \exp\left(i\omega(t - r/v)\right)$$

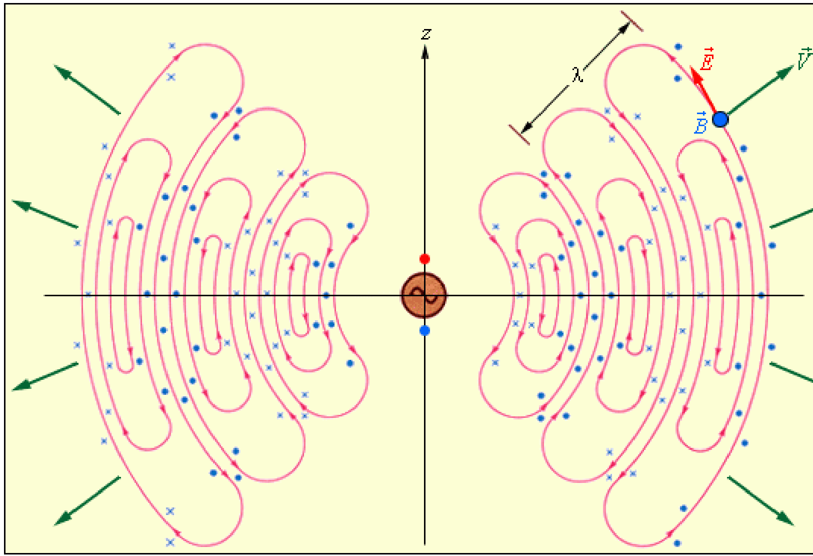
$$\mathbf{S} = [\mathbf{E} \times \mathbf{H}]$$

$$S_r = \mathrm{Re}\{E_r\}\mathrm{Re}\{H_\alpha\} = \frac{\omega^4 p_0^2 \sin^2\theta}{16\pi^2 \varepsilon\varepsilon_0 r^2 v^3} \cos^2\left(\omega(t - r/v)\right)$$

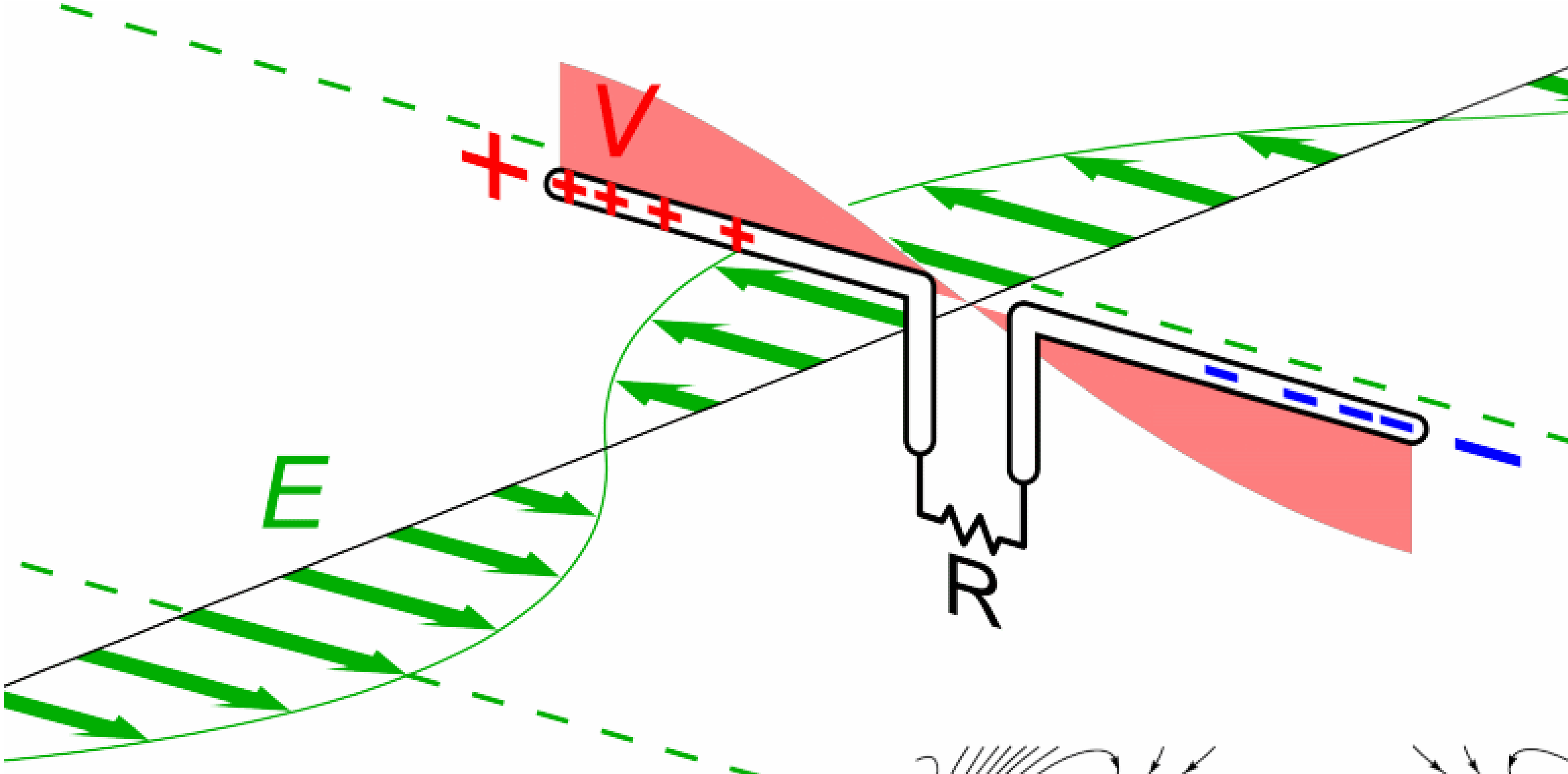
$$< S_r > = \frac{\omega^4 p_0^2 \sin^2\theta}{16\pi^2 \varepsilon\varepsilon_0 r^2 v^3} < \cos^2\left(\omega(t - r/v)\right) > = \boxed{\frac{\omega^4 p_0^2 \sin^2\theta}{32\pi^2 \varepsilon\varepsilon_0 r^2 v^3}}$$

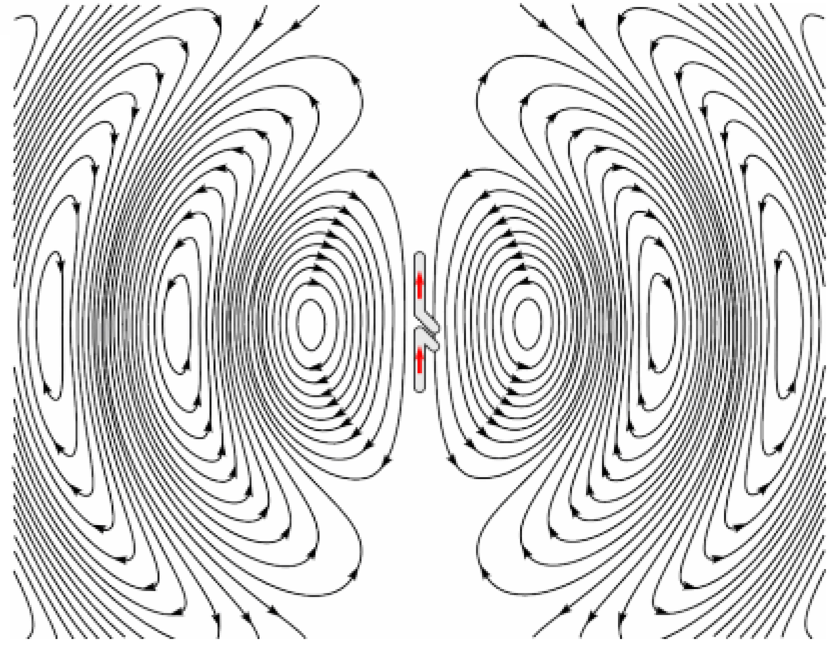


120
100
80
60
40
20
0
0
20
40
60
80
100
120



z
λ
$\vec{E}$
$\vec{V}$
$\vec{B}$


V
+
E
R



$$< S_r >= \frac{\omega^4 p_0^2 \sin^2\theta}{16\pi^2\varepsilon\varepsilon_0 r^2 v^3} < \cos^2\left(\omega(t - r/v)\right) >= \boxed{\frac{\omega^4 p_0^2 \sin^2\theta}{32\pi^2\varepsilon\varepsilon_0 r^2 v^3}}$$

Поток электромагнитной энергии сквозь поверхность сферы радиуса r

$$P = \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\varphi r^2 < S_r > = \frac{\omega^4 p_0^2}{32\pi^2\varepsilon\varepsilon_0 v^3} \int_0^{2\pi} d\varphi \int_0^{\pi} \sin^3\theta d\theta =$$

$$\cos\theta = x$$

$$\boxed{= \frac{\omega^4 p_0^2}{12\pi\varepsilon\varepsilon_0 v^3}}$$

$$\int_0^{\pi} \sin^3\theta d\theta = \int_1^{-1} (x^2 - 1) dx = -\frac{2}{3} - (-2) = \frac{4}{3}$$

Закон сохранения импульса

$$\frac{d\mathbf{p}_0}{dt} = q_0\{\mathbf{E} + [\mathbf{v}\times\mathbf{B}]\} \qquad n\frac{d\mathbf{p}_0}{dt} = nq_0\{\mathbf{E} + [\mathbf{v}\times\mathbf{B}]\}$$

$$\frac{d\mathbf{p}}{dt} = \oint_V \rho(\mathbf{r},t)\{\mathbf{E} + [\mathbf{v}\times\mathbf{B}]\}d\mathbf{r}$$

$$\rho = \operatorname{div}\mathbf{D} \qquad \mathbf{j} = \rho\mathbf{v} = \operatorname{rot}\mathbf{H} - \frac{\partial\mathbf{D}}{\partial t}$$

$$\frac{d\mathbf{p}}{dt} = \oint_V \mathbf{E}\operatorname{div}\mathbf{D}dv + \oint_V [\operatorname{rot}\mathbf{H}\times\mathbf{B}]dv - \oint_V [\partial\mathbf{D}/\partial t\times\mathbf{B}]dv +$$

$$+ \oint_V [\underbrace{(\operatorname{rot}\mathbf{E} + \partial\mathbf{B}/\partial t)}_{=0}\times\mathbf{D}]dv + \oint_V \mathbf{H}\underbrace{\operatorname{div}\mathbf{B}}_{=0}dv \quad \Rightarrow$$

$$\Rightarrow \frac{d\mathbf{p}}{dt} = \oint_V \mathbf{E}\,\mathrm{div}\mathbf{D}dv + \oint_V [\mathrm{rot}\,\mathbf{H}\times\mathbf{B}]dv - \oint_V [\partial\mathbf{D}/\partial t\times\mathbf{B}]dv +$$

$$+\oint_V [(\mathrm{rot}\,\mathbf{E} + \partial\mathbf{B}/\partial t)\times\mathbf{D}]dv + \oint_V \mathbf{H}\,\mathrm{div}\mathbf{B}dv$$

$$\frac{d\mathbf{p}}{dt} = \oint_V \underline{(\mathbf{E}\,\mathrm{div}\,\mathbf{D} - [\mathbf{D}\times\mathrm{rot}\,\mathbf{E}])}dv + \oint_V (\mathbf{H}\,\mathrm{div}\,\mathbf{B} - \underline{[\mathbf{B}\times\mathrm{rot}\,\mathbf{H}])}dv -$$

$$-\oint_V [\partial\mathbf{D}/\partial t\times\mathbf{B}]dv - \oint_V [\mathbf{D}\times\partial\mathbf{B}/\partial t]dv$$

$$\oint_V (\mathbf{E}\,\mathrm{div}\,\mathbf{D} - [\mathbf{D}\times\mathrm{rot}\,\mathbf{E}])dv = \varepsilon\varepsilon_0 \oint_V (\mathbf{E}\,\mathrm{div}\,\mathbf{E} - [\mathbf{E}\times\mathrm{rot}\,\mathbf{E}])dv \Rightarrow$$

$$\Rightarrow = 0$$

$$\mathbf{E}\,\mathrm{div}\,\mathbf{E} = (\vec{\nabla}\cdot\mathbf{E})\mathbf{E} - (\mathbf{E}\cdot\vec{\nabla})\mathbf{E}$$

Воспользуемся двумя формулами

$$[\mathbf{E}\cdot\mathrm{rot}\,\mathbf{E}] = \frac{1}{2}\mathrm{grad}\,\mathbf{E}^2 - (\mathbf{E}\cdot\vec{\nabla})\mathbf{E}$$

$$\mathbf{E}\,\mathrm{div}\,\mathbf{E} = (\vec{\nabla}\cdot\mathbf{E})\mathbf{E} - (\mathbf{E}\cdot\vec{\nabla})\mathbf{E}$$

x-ая компонента

$$E_x\partial_x E_x + E_x\partial_y E_y + E_x\partial_z E_z =$$

$$= \partial_x E_x^2 + \partial_y E_y E_x + \partial_z E_z E_x - E_x\partial_x E_x - E_y\partial_y E_x - E_z\partial_z E_x$$

$$[\mathbf{E}\times\mathrm{rot}\,\mathbf{E}] = \frac{1}{2}\mathrm{grad}\,\mathbf{E}^2 - (\mathbf{E}\cdot\vec{\nabla})\mathbf{E}$$

x-ая компонента

$$E_y(\mathrm{rot}\,\mathbf{E})_z - E_z(\mathrm{rot}\,\mathbf{E})_y = E_x\partial_x E_x + E_y\partial_x E_y + E_z\partial_x E_z -$$

$$-(E_x\partial_x + E_y\partial_y + E_z\partial_z)E_x$$

$$E_y\partial_x E_y - E_y\partial_y E_x + E_z\partial_x E_z - E_z\partial_z E_x =$$

$$= E_x\partial_x E_x + E_y\partial_x E_y + E_z\partial_x E_z - E_x\partial_x E_x - E_y\partial_y E_x - E_z\partial_z E_x$$

$$\mathbf{E}\,\mathrm{div}\,\mathbf{E} - [\mathbf{E}\times\mathrm{rot}\,\mathbf{E}] = (\vec{\nabla}\cdot\mathbf{E})\mathbf{E} - \frac{1}{2}\vec{\nabla}\mathbf{E}^2$$

$$\mathbf{E}\,\mathrm{div}\,\mathbf{E}-[\mathbf{E}\times\mathrm{rot}\,\mathbf{E}]=(\vec{\nabla}\cdot\mathbf{E})\mathbf{E}-\frac{1}{2}\vec{\nabla}\mathbf{E}^2$$

$$\oint_V\left(\mathbf{E}\,\mathrm{div}\,\mathbf{E}-[\mathbf{E}\times\mathrm{rot}\,\mathbf{E}]\right)dv=\oint_V\{(\vec{\nabla}\cdot\mathbf{E})\mathbf{E}-\frac{1}{2}\vec{\nabla}\mathbf{E}^2\}dv=$$

$$E\sim\frac{1}{r^2}$$

$$=\oint_S(\mathbf{n}\cdot\mathbf{E})\mathbf{E}dS-\frac{1}{2}\oint_S\mathbf{n}\cdot\mathbf{E}^2dS=0$$

$$\oint_V\vec{\nabla}\psi dv=\oint_S\mathbf{n}\psi dS$$

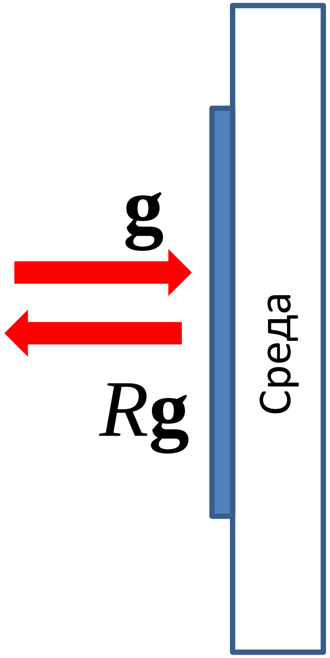
$$\frac{d\mathbf{p}}{dt}=\oint_V\left(\underbrace{\cancel{\mathbf{E}\,\mathrm{div}\,\mathbf{D}-[\mathbf{D}\times\mathrm{rot}\,\mathbf{E}]}}_{=0}\right)dv+\oint_V\left(\underbrace{\cancel{\mathbf{H}\,\mathrm{div}\,\mathbf{B}-[\mathbf{B}\times\mathrm{rot}\,\mathbf{H}]}}_{=0}\right)dv$$

$$-\oint_V[\partial\mathbf{D}/\partial t\times\mathbf{B}]dv-\oint_V[\mathbf{D}\times\partial\mathbf{B}/\partial t]dv=-\frac{\partial}{\partial t}\oint_V[\mathbf{D}\times\mathbf{B}]dv$$

$$\boxed{\frac{d\mathbf{p}}{dt}+\frac{\partial}{\partial t}\oint_V[\mathbf{D}\times\mathbf{B}]dv=0\qquad\mathbf{g}=[\mathbf{D}\times\mathbf{B}]}$$

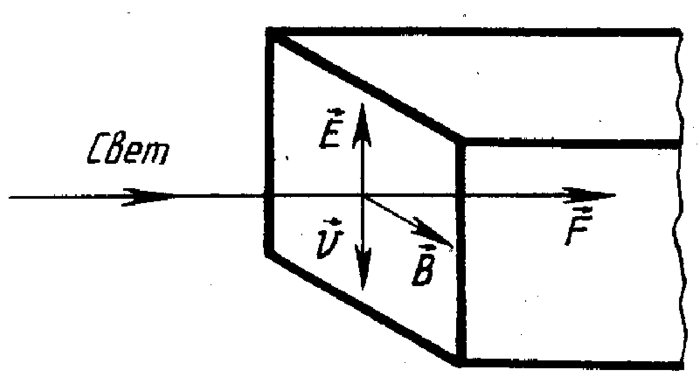
- плотность импульса электромагнитного поля

$$\mathbf{g} = [\mathbf{D} \times \mathbf{B}] = \varepsilon\varepsilon_0\mu\mu_0[\mathbf{E} \times \mathbf{H}] = \mathbf{S}_p / v^2$$

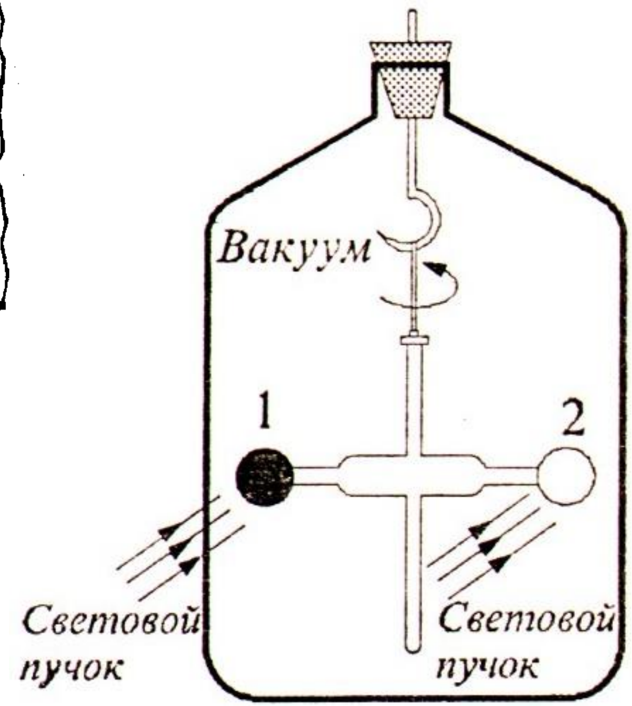


$$(1+R)g\Delta S v \Delta t = p\Delta S\Delta t$$

$$p = (1+R)gv = (1+R)S_p / v$$



П.Н. Лебедев 1900



- 10^{14} Па - давление в центре взрыва водородной бомбы;
- 10^{13} Па - давление в центре Земли;
- $3 \cdot 10^{9}$ Па - давление колеса вагона на рельсы;
- $5 \cdot 10^{7}$ Па - давление жала пчелы;
- 10^{6} Па - давление конькобежца на лед;
- $4 \cdot 10^{5}$ Па - давление человека при ходьбе;
- 10^{-8} Па - давление воздуха на высоте 800 км.

Давление от лазерной указки 10^{-6} Па

Квазистационарное приближение

$$\operatorname{rot}\mathbf{E}(\mathbf{r},t) = -\frac{\partial\mathbf{B}(\mathbf{r},t)}{\partial t}$$

$$\operatorname{rot}\mathbf{H}(\mathbf{r},t) = \mathbf{j}(\mathbf{r},t) + \cancel{\frac{\partial\mathbf{D}(\mathbf{r},t)}{\partial t}} = 0$$

$$\operatorname{div}\mathbf{D}(\mathbf{r},t) = \rho(\mathbf{r},t)$$

$$\operatorname{div}\mathbf{B}(\mathbf{r},t) = 0$$

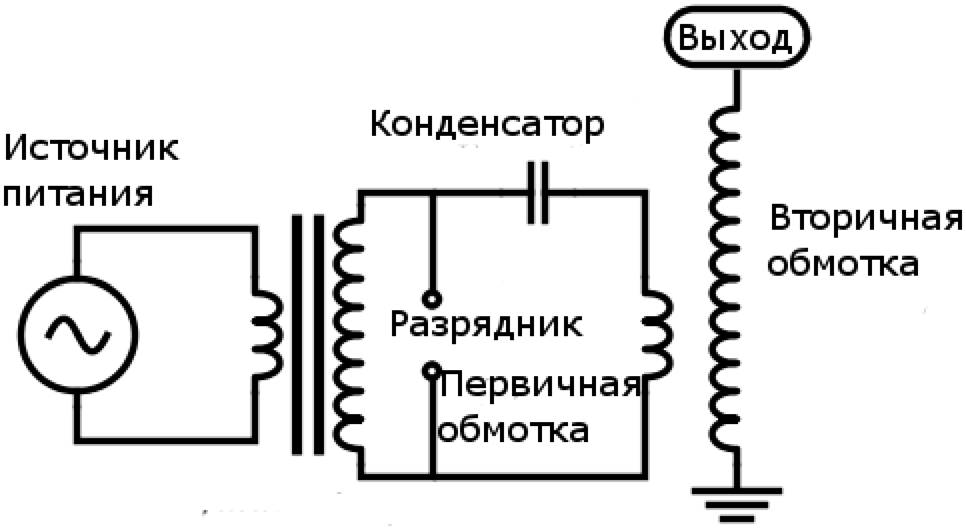
$$\mathbf{E} = \mathbf{E}_0 \sin\omega t \qquad \frac{\partial\mathbf{D}}{\partial t} = \varepsilon\varepsilon_0\omega\mathbf{E}_0\cos\omega t \qquad \mathbf{j} = \sigma\mathbf{E} = \sigma\mathbf{E}_0\sin\omega t$$

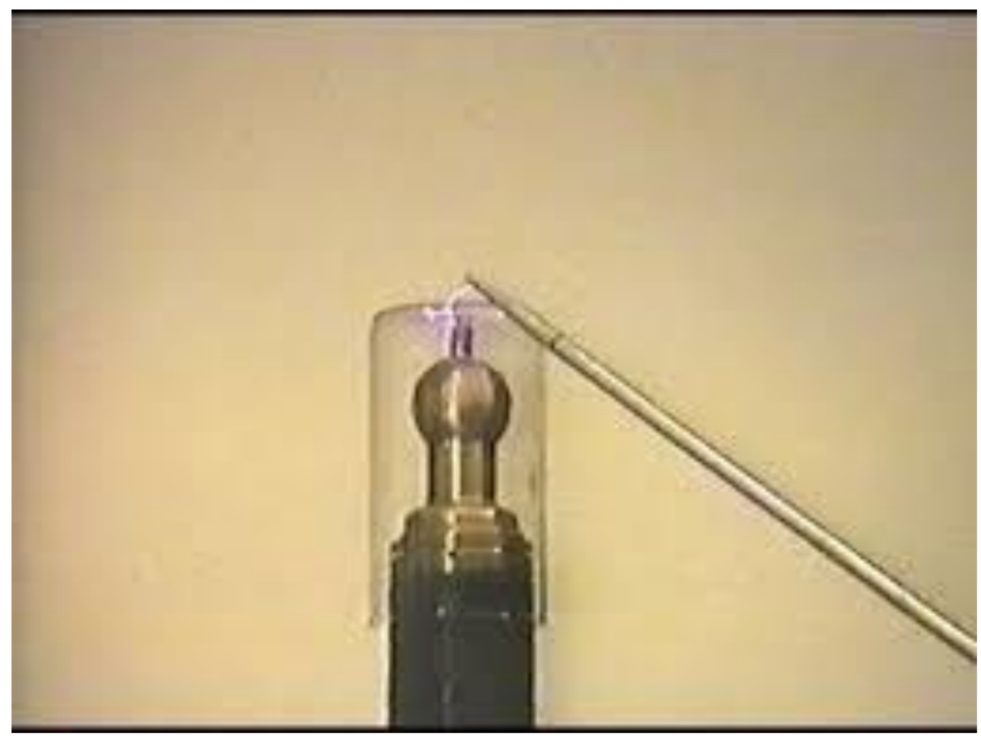
$$\mathbf{j}(\mathbf{r},t) >> \frac{\partial\mathbf{D}(\mathbf{r},t)}{\partial t} \Longleftrightarrow \sigma\mathbf{E}_0 >> \varepsilon\varepsilon_0\omega\mathbf{E}_0 \Rightarrow \sigma >> \varepsilon\varepsilon_0\omega$$

Для меди $\omega << 10^{17}\ \text{c}^{-1}$

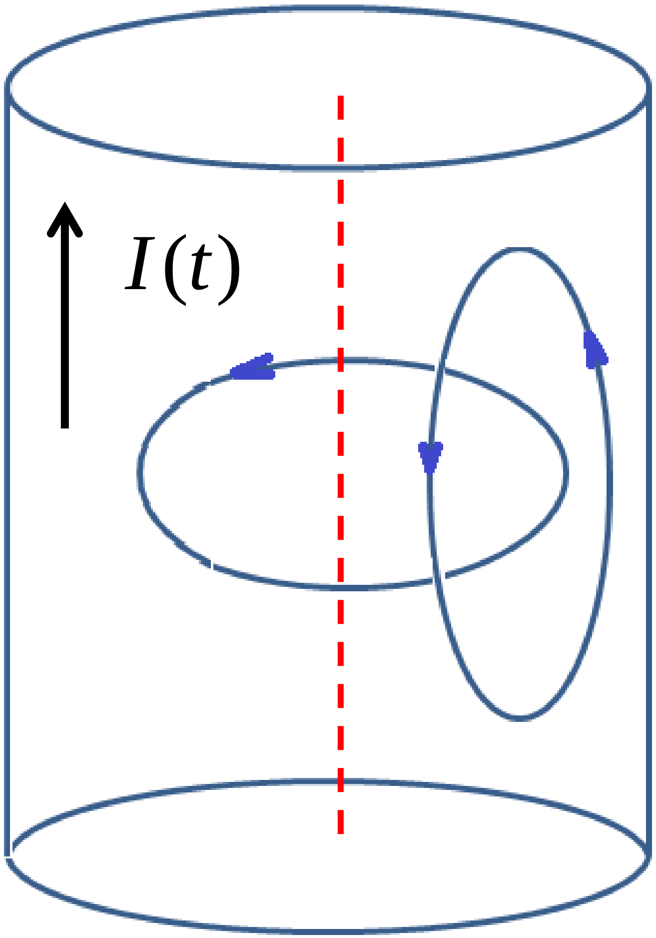
Току, меняющемуся с частотой 50 Гц $\Rightarrow$ $\lambda \approx 600\ \text{км} \quad >> \mathrm{L}$

$$\operatorname{rot}\mathbf{H}(\mathbf{r},t) = \mathbf{j}(\mathbf{r},t) + \frac{\partial\mathbf{D}(\mathbf{r},t)}{\partial t}$$





Скин-эффект



$$\frac{dI(t)}{dt} > 0 \quad \mathrm{rot}\,\mathbf{E}(\mathbf{r},t) = -\frac{\partial \mathbf{B}(\mathbf{r},t)}{\partial t}$$

$$\mathrm{rot}\,\mathbf{H}(\mathbf{r},t) = \mathbf{j}(\mathbf{r},t) = \sigma \mathbf{E}$$

$$\mathrm{rotrot}\,\mathbf{E}(\mathbf{r},t) = -\mathrm{rot}\,\frac{\partial \mathbf{B}(\mathbf{r},t)}{\partial t} =$$

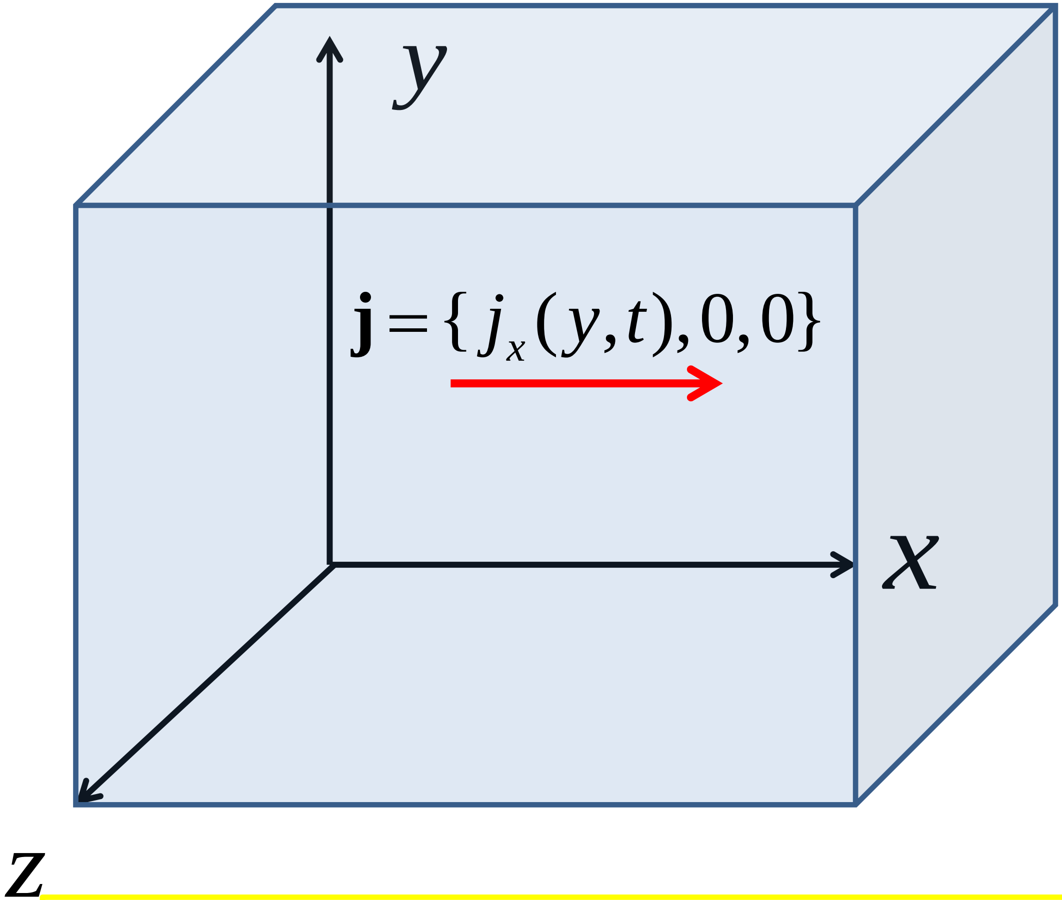
$$= -\frac{\partial}{\partial t}\mathrm{rot}\,\mathbf{B}(\mathbf{r},t) = -\mu\mu_0 \frac{\partial}{\partial t}\mathrm{rot}\,\mathbf{H}(\mathbf{r},t) =$$

$$= -\mu\mu_0\sigma \frac{\partial \mathbf{E}(\mathbf{r},t)}{\partial t}$$

$$\mathrm{rotrot}\,\mathbf{E}(\mathbf{r},t) = \mathrm{graddiv}\,\mathbf{E}(\mathbf{r},t) - \Delta \mathbf{E}(\mathbf{r},t)$$

$$\mathrm{div}\,\mathbf{D} = 0 \qquad = 0$$

$$\Delta \mathbf{E} = \mu\mu_0\sigma \frac{\partial \mathbf{E}}{\partial t}$$



$$\mathbf{E} = \{E_x(y,t), 0, 0\}$$

$$E_x(y,t) = E_0(y)\exp(i\omega t)$$

$$\Delta \mathbf{E} = \mu\mu_0\sigma \frac{\partial \mathbf{E}}{\partial t}$$

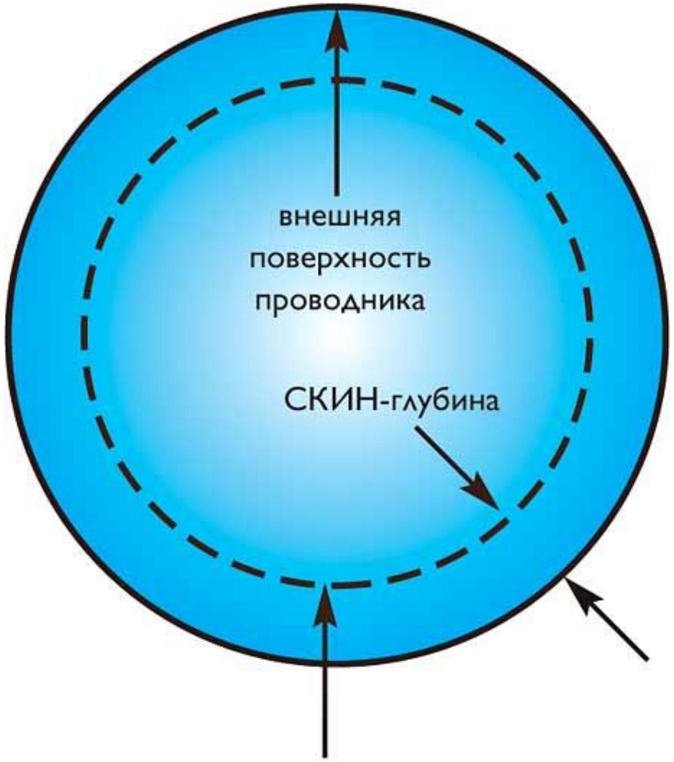
$$\Downarrow$$

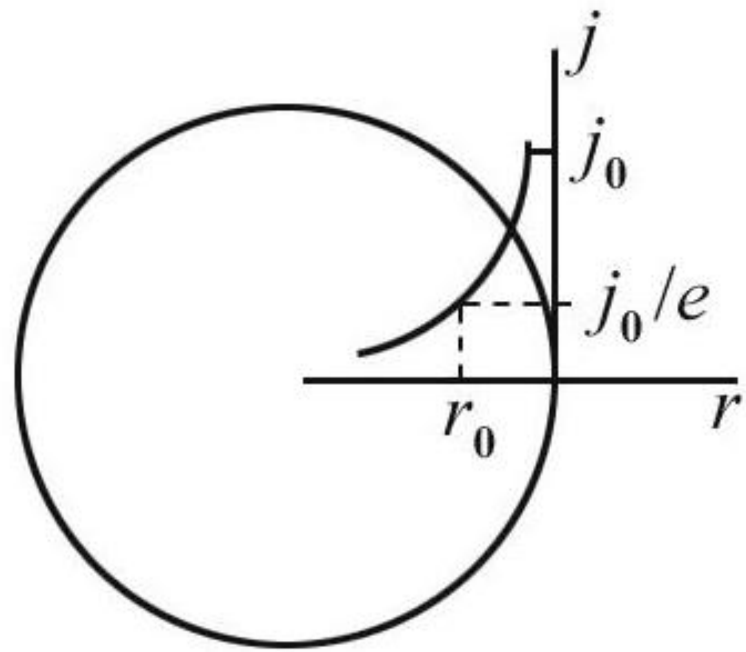
$$\frac{d^2 E_0}{dy^2} = i\mu\mu_0\sigma\omega E_0$$

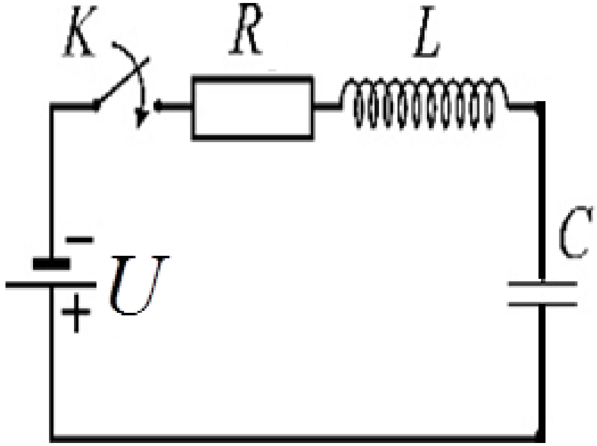
$$E_0 = A\exp[-\alpha(1+i)y] \quad \text{где} \quad \alpha = (\mu\mu_0\omega\sigma/2)^{1/2}$$

$j_x = j_0 \exp[-\alpha(1+i)y]$ $\quad d = 1/\alpha$ - толщина скин-слоя

Для меди при $\omega = 10^4\,\text{с}^{-1}$ $d = 4$ мм







$$U - L\frac{dI}{dt} = IR + \frac{q}{C} \qquad I = \frac{dq}{dt}$$

$$q(0) = 0 \qquad I(0) = 0$$

$$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{q}{LC} = \frac{U}{L}$$

$$\frac{d^2q}{dt^2}+\frac{R}{L}\frac{dq}{dt}+\frac{q}{LC}=\frac{U}{L}$$

$$q(t)=CU+A\exp\left(-\frac{R}{2L}t\right)\sin\left(\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}\times t+\phi\right)$$

$$I(t)=\frac{dq}{dt}=A\exp\left(-\frac{R}{2L}t\right)\left\{-\frac{R}{2L}\sin\left(\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}\times t+\phi\right)+\right.$$

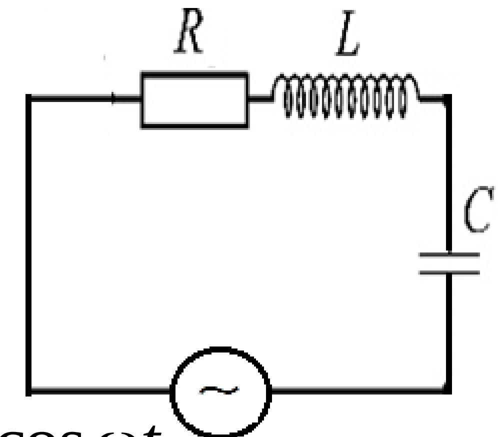
$$\left.+\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}\cos\left(\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}\times t+\phi\right)\right\}$$

$$0=CU+A\sin\phi \qquad 0=\left(-\frac{R}{2L}\right)\sin\phi+\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}\cos\phi$$

$$A=-\frac{CU}{\sqrt{1-CR^2/4L}} \qquad \phi=\operatorname{arctg}\sqrt{\frac{4L}{CR^2}-1}$$

$$q(t)=CU\left[1-\frac{\exp\left(-Rt/2L\right)}{\sqrt{1-CR^2/4L}}\sin\left(\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}\times t+\operatorname{arctg}\sqrt{\frac{4L}{CR^2}-1}\right)\right]$$

Метод комплексных амплитуд



$U_0\cos\omega t$

$$\frac{d^2q}{dt^2}+\frac{R}{L}\frac{dq}{dt}+\frac{q}{LC}=\frac{U_0}{L}\cos\omega t$$

$$L\frac{d^2I}{dt^2}+R\frac{dI}{dt}+\frac{I}{C}=-\omega U_0\sin\omega t$$

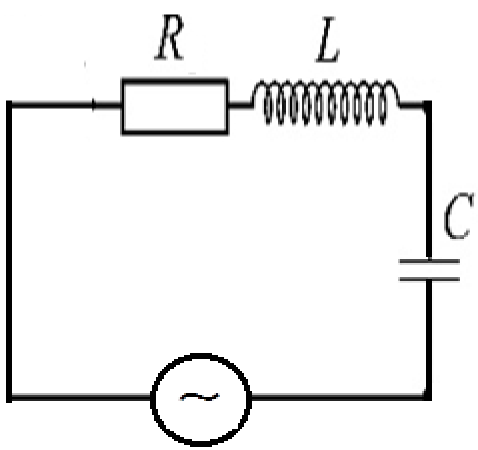
$$U_0\cos\omega t=U_0\operatorname{Re}\{\exp(i\omega t)\}\qquad I(t)=\operatorname{Re}\{\hat{I}\exp(i\omega t)\}$$

$$\operatorname{Re}\left\{\left[L\frac{d^2}{dt^2}+R\frac{d}{dt}+\frac{1}{C}\right]\hat{I}\exp(i\omega t)\right\}=U_0\operatorname{Re}\{i\omega\exp(i\omega t)\}$$

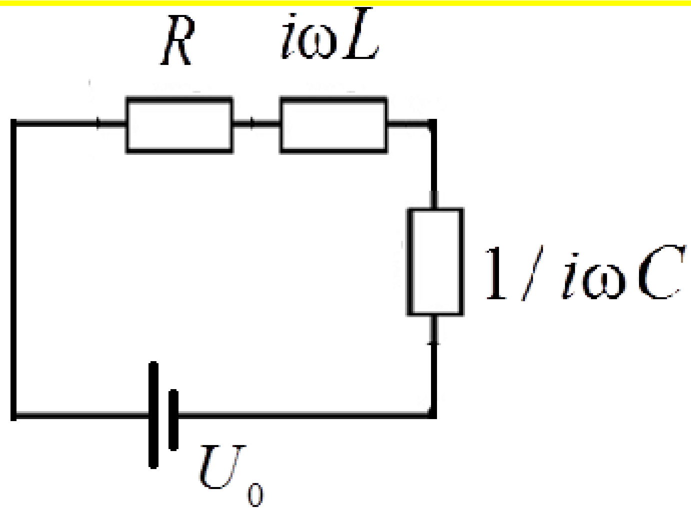
$$\left[L\frac{d^2}{dt^2}+R\frac{d}{dt}+\frac{1}{C}\right]\hat{I}\exp(i\omega t)=U_0\{i\omega\exp(i\omega t)\}$$

$$\left[-\omega^2 L+i\omega R+\frac{1}{C}\right]\hat{I}=i\omega U_0 \qquad \left[i\omega L+R+\frac{1}{i\omega C}\right]\hat{I}=U_0$$

$$L\Rightarrow Z_L=i\omega L \quad R\Rightarrow Z_R=R \quad C\Rightarrow Z_C=1/i\omega C$$



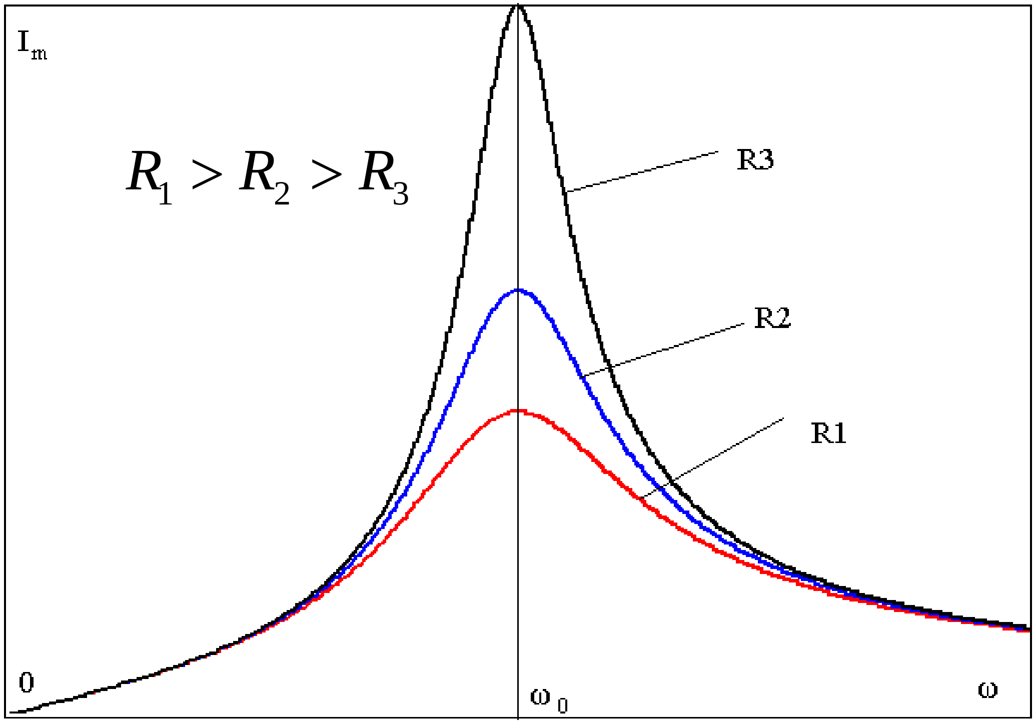
$U_0\cos\omega t$



$$\left[Z_L+Z_R+Z_C\right]\hat{I}=U_0 \qquad \hat{I}=\frac{U_0}{Z_L+Z_R+Z_C}$$

$$I(t) = \mathrm{Re}\left\{\hat{I}\exp(i\omega t)\right\} = \mathrm{Re}\left\{\frac{U_0 \exp(i\omega t)}{i\omega L + R + 1/i\omega C}\right\}$$

$$I(t) = \frac{U_0}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \cos\left(\omega t - \mathrm{arctg}\left(\frac{\omega L - 1/\omega C}{R}\right)\right)$$



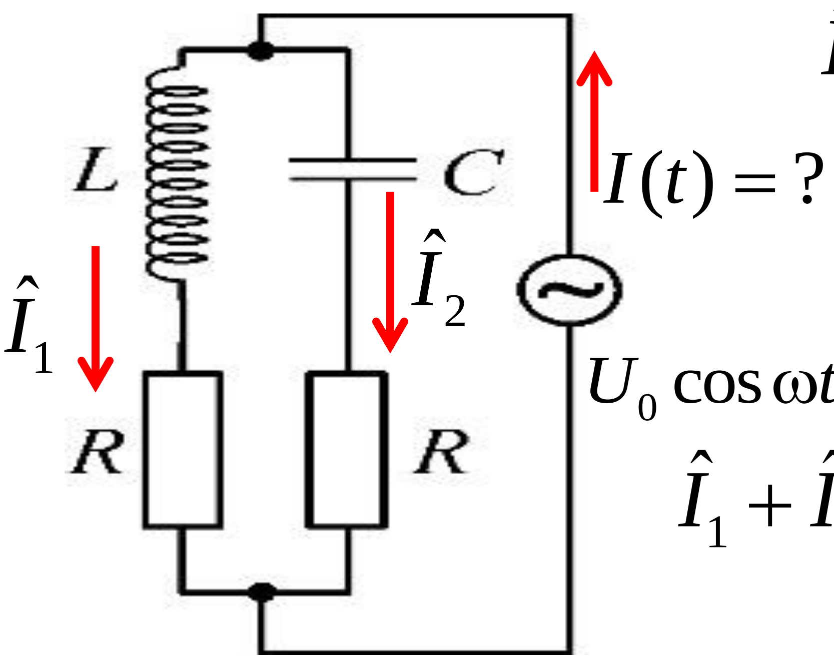
$$I_m = \frac{U_0}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

$$\Phi = \mathrm{arctg}\left(\frac{\omega L - 1/\omega C}{R}\right)$$

$$\omega_0 \approx \omega_k = \sqrt{1/LC}$$

$$\Phi(\omega_m \approx \omega_0) = 0$$

Законы Кирхгофа и метод комплексных амплитуд



$$\hat{I}_1 = \frac{U_0}{R + i\omega L} \qquad \hat{I}_2 = \frac{U_0}{R + 1/i\omega C}$$

$$I(t) = \mathrm{Re}\left\{(\hat{I}_1 + \hat{I}_1)\exp(i\omega t)\right\}$$

$$\hat{I}_1 + \hat{I}_2 = U_0 \frac{2R + i(\omega L - 1/\omega C)}{(R + i\omega L)(R + 1/i\omega C)}$$

$$I(t) = U_0 \frac{\sqrt{4R^2 + (\omega L - 1/\omega C)^2}}{\sqrt{(R^2 + L/C)^2 + R^2(\omega L - 1/\omega C)^2}} \times$$

$$\times \cos\left\{\omega t - \mathrm{arctg}(\omega L / R) + \mathrm{arctg}(1/\omega RC) + \mathrm{arctg}\left(\frac{\omega L - 1/\omega C}{2R}\right)\right\}$$